Extra Dimensions for TeV Physics

Parma International School of Theoretical Physics
"Theoretical Tools for the LHC"
Parma, August 31-September 4, 2009

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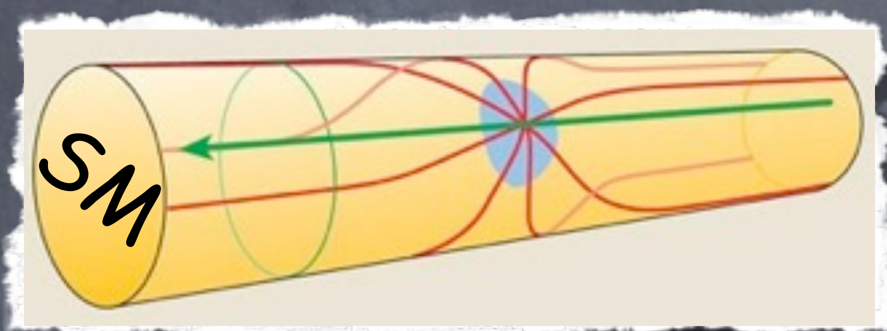
Why Extra Dimensions?

- Why not? Or actually, why only 4 dimensions?
  (may be: Gauss law for gravity and e.m.; renormalizability;...)
- Extra dimensions can actually be quite useful
  - unification of fundamental interactions:
    - old Kaluza-Klein idea: 5D gravity=4D gravity + $U(1)_{em}$?
    - quantization of gravity: superstrings need extra dimensions
  - hierarchy problem, i.e., why is gravity so weak
    - large (mm size) extra dimensions
    - warped extra dimensions
  - symmetry breaking by orbifold compactification or boundary conditions
  - dynamical generation of fermion mass hierarchy + flavour structure
  - dark matter particles; inflation; accelerated expansion...
- Tools to study strongly coupled systems
  - technicolor/composite Higgs models
  - QCD
  - plasma, condense matter systems (superconductors, vortices...)

Extra Dimensions for TeV/LHC Physics

- Hierarchy problem, i.e., why is gravity so weak
  - large (mm size) extra dimensions
    - gravity is diluted into space while we are localized on a brane

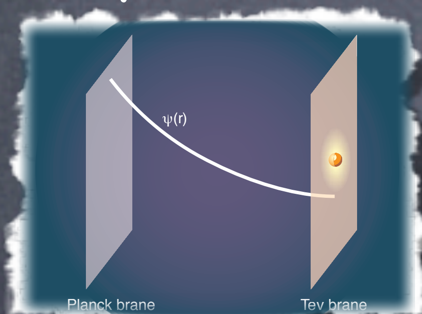


$$d^{4+n}x\ \sqrt{|g_{4+n}|}\ M^{2+n}\mathcal{R} = d^4x\ \sqrt{|g_4|}\ M_{Pl}^2\mathcal{R}$$

$$M_{Pl}^2 = V_n M^{2+n}$$

$M_{Pl} = 10^{19}$ GeV $\quad M_* = 1$ TeV $\quad V_2 = (2 \text{ mm})^2$

  - warped extra dimensions
    - gravity is localized away from SM matter and we feel only the tail of the graviton



graviton wavefunction is exponentially localized away from SM brane

$$v = M_\star e^{-\pi R M_\star}$$

$M_* = 10^{19}$ GeV $\quad v = 1$ TeV $\quad R \sim 11/M_*$

- Fermion mass hierarchy & flavour structure

  fermion profiles:

  the bigger overlap with Higgs vev, the bigger the mass

- EW symmetry breaking

  Orbifold breaking, Higgsless

Disclaimer & References

I will introduce the notion of Kaluza-Klein decomposition but
I won't describe in details the various incarnations of extra dimensions
nor present the collider signatures and discuss the constraints

- Flat
  - small (MPlanck/GUT size)
    - Kaluza-Klein
    - string/sugra compactications
    - GUT orbifold breaking
  - intermediate (TeV size)
    - universal extra dimensions (UED)
    - constrained standard model
    - gauge-Higgs unification
  - large (mm size)
    - Arkani-Hamed Dvali Dimopoulos
  - infinite
    - Dvali Gabadadze Porrati
  - discrete
    - Little Higgs
- Curved/Warped
  - Randall-Sundrum
    - RS_1
    - RS_2
    - Susy RS
    - GUT RS
  - Higgsless
  - Composite Higgs
  - Gaugephobic

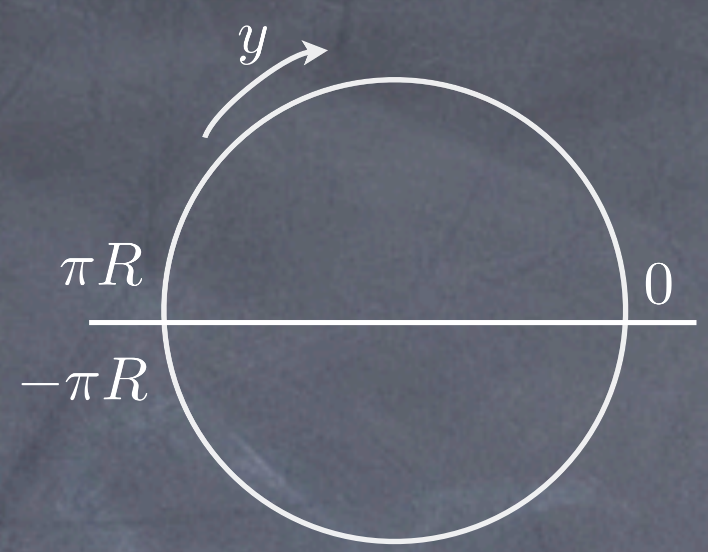
Disclaimer & References

(Partial) list of references to learn more about extra dimensions

- Csaki hep-ph/0404096 @ TASI'02
- Gabadadze hep-ph/0308112 @ ICTP'02
- Rattazzi hep-ph/0607055 @ Cargèse'03
- Kribs hep-ph/0605325, Sundrum hep-th/0508134 @ TASI'04
- Rizzo hep-ph/0409309 @ SLAC Summer Institute'04
- Gherghetta hep-ph/0601213, Grojean, Hewett, Rubakov hep-ph/0104152
  @ Les Houches'05 (slides at http://lavignac.home.cern.ch/lavignac/Houches/)
- Agashe @ TASI'06 (slides at http://quark.phy.bnl.gov/~dawson/agashe_1.pdf)
- Dobrescu @ TASI'08 (slides at http://physicslearning2.colorado.edu/tasi/tasi_2008/tasi_2008.htm)
- Dvali @ Zuoz'08 (slides at http://ltpth.web.psi.ch/zuoz_school/)
- Cheng, Gherghetta @ TASI'09
  (slides at http://physicslearning2.colorado.edu/tasi/tasi_2009/tasi_2009.htm)
- Shifman hep-ph/0907.3074

KK decomposition

Compactification on a Circle: real scalar field



circle = $\mathbb{R}/T_{2\pi R}$

$$y \sim y + 2\pi R$$

the fields at y and y+2πR should be equal

y=coordinate along the extra dimension
x=usual 4D coordinates

$$\phi(y,x) = \phi(y + 2\pi R, x)$$

the 5D fields can be decomposed
in Fourier modes
= Kaluza-Klein modes

$$\phi(y,x) = \sum_n \mathcal{N}_n^+ \cos\left(\frac{ny}{R}\right) \phi_n^+(x) + \mathcal{N}_n^- \sin\left(\frac{ny}{R}\right) \phi_n^-(x)$$

wavefunction =
localization of KK mode
along the xdim

4D
Kaluza-Klein modes

the coefficients $\mathcal{N}_n^\pm$ are fixed by requiring a canonical normalization of the 4D KK modes

Compactification on a Circle: real scalar field

$$\phi(y,x) = \sum_n \mathcal{N}_n^+ \cos\left(\frac{ny}{R}\right) \phi_n^+(x) + \mathcal{N}_n^- \sin\left(\frac{ny}{R}\right) \phi_n^-(x)$$

5D Lagrangian ⇨ 4D Lagrangian for KK modes

$$\mathcal{S} = \int d^4x \int_{-\pi R}^{\pi R} dy \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}(\partial_5\phi)^2 - \frac{1}{2}m_{5D}^2\phi^2 \right)$$

(+----) metric

$[\phi]$=mass$^{3/2}$

4D kinetic terms   4D mass term

$$\int_{-\pi R}^{\pi R} dy \cos\left(\frac{ny}{R}\right) \cos\left(\frac{my}{R}\right) = 2^{\delta_{n0}}\pi R\, \delta_{mn} \quad ⇨ \quad \mathcal{N}_n^+ = \frac{1}{\sqrt{2^{\delta_{n0}}\pi R}}$$

$[\mathcal{N}_n^\pm]$=mass$^{1/2}$

$$\int_{-\pi R}^{\pi R} dy \sin\left(\frac{ny}{R}\right) \sin\left(\frac{my}{R}\right) = \pi R\, \delta_{mn} \quad ⇨ \quad \mathcal{N}_n^- = \frac{1}{\sqrt{\pi R}}$$

$[\phi_n]$=mass

$$\mathcal{S} = \int d^4x \sum_{n=0}^{\infty} \left(\frac{1}{2}\partial_\mu\phi_n^+\partial^\mu\phi_n^+ - \frac{1}{2}\left(m_{5D}^2 + \frac{n^2}{R^2}\right){\phi_n^+}^2 \right) + \sum_{n=1}^{\infty} \left(\frac{1}{2}\partial_\mu\phi_n^-\partial^\mu\phi_n^- - \frac{1}{2}\left(m_{5D}^2 + \frac{n^2}{R^2}\right){\phi_n^-}^2 \right)$$

⋮ ⋮

$m_{5D}^2 + 9/R^2$

$m_{5D}^2 + 4/R^2$

$m_{5D}^2 + 1/R^2$

m_{5D}^2

+ states   - states

5D field=infinite tower of massive 4D fields

depending of the energy available, you can probe more and more of these KK modes

Compactification on a Circle: real scalar field

let us introduce a complex notation that will simplify the computations once interactions are introduced

complex linear combinations

$$\phi_n = \frac{1}{\sqrt{2}}\left(\phi_n^+ - i\phi_n^-\right)$$

$$\phi_{-n} = \phi_n^\dagger$$

$$\phi_0 = \phi^\dagger = \phi_0^+$$

$$\phi(y,x) = \sum_{n=-\infty}^{+\infty} \frac{1}{\sqrt{2\pi R}} e^{iny/R}\phi_n(x)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_{5D}^2\phi_0^2 + \sum_{n=1}^{\infty}\left(\partial_\mu\phi_n\partial^\mu\phi_{-n} - \left(m_{5D}^2 + \frac{n^2}{R^2}\right)\phi_n\phi_{-n}\right)$$

KK number conservation = conversation of momentum along 5^{th} dimension

Let us introduce interactions, e.g. ϕ^4

$$\int_{-\pi R}^{\pi R} dy\, \lambda_{5D}\phi^4$$

$[\phi]$=mass$^{3/2}$

$[\lambda_{5D}]$=mass^{-1}

$$= \sum_{m,n,p,q=-\infty}^{\infty} \frac{\lambda_{5D}}{(2\pi R)^2} \underbrace{\int_{-\pi R}^{\pi R} dy\, e^{i(m+n+p+q)y/R}}_{2\pi R\delta_{m+n+p+q}} \phi_m(x)\phi_n(x)\phi_p(x)\phi_q(x)$$

$$= \sum_{m+n+p+q=0} \frac{\lambda_{5D}}{2\pi R}\phi_m(x)\phi_n(x)\phi_p(x)\phi_q(x) = \frac{\lambda_{5D}}{2\pi R}\left(\phi_0^4 + 2\sum_{n=1}^{\infty}\phi_0^2\phi_n\phi_{-n}\right)$$

Compactification on a Circle: gauge field

complex notation

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{iny/R}\, A_M^{(n)}(x) \qquad M = (\mu = 0\ldots 3, 5)$$

$$\int d^4x\, dy \left(-\frac{1}{4} F_{MN} F^{MN}\right)$$

$$= \int d^4x\, dy \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu 5} F^{\mu 5}\right)$$

$$= \int d^4x\, dy \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left(\partial_\mu A_5 - \partial_5 A_\mu\right)\left(\partial^\mu A_5 - \partial_5 A^\mu\right)\right)$$

focusing on "kinetic piece"

(+----) metric

$$= \int d^4x \sum_{n=-\infty}^{+\infty} \left(-\frac{1}{4} F_{\mu\nu}^{(n)} F^{(-n)\,\mu\nu} + \frac{1}{2}\left(\partial_\mu A_5^{(n)} - \frac{in}{R} A_\mu^{(n)}\right)\left(\partial^\mu A_5^{(-n)} + \frac{in}{R} A^{(-n)\,\mu}\right)\right)$$

gauge transformation (n≠0)

$$A_\mu^{(n)} \to \tilde{A}_\mu^{(n)} = A_\mu^{(n)} - \frac{i}{n/R} \partial_\mu A_5^{(n)}$$

$$F_{\mu\nu}^{(n)} \to \tilde{F}_{\mu\nu}^{(n)} = F_{\mu\nu}^{(n)}$$

$$= \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^{(0)} F^{(0)\,\mu\nu} + \frac{1}{2} \partial_\mu A_5^{(0)} \partial^\mu A_5^{(0)} + 2 \sum_{n=1}^{\infty} \left(\tilde{F}_{\mu\nu}^{(n)} \tilde{F}^{(-n)\,\mu\nu} + \frac{n^2}{2R^2} \tilde{A}_\mu^{(n)} \tilde{A}^{(-n)\,\mu}\right)\right)$$

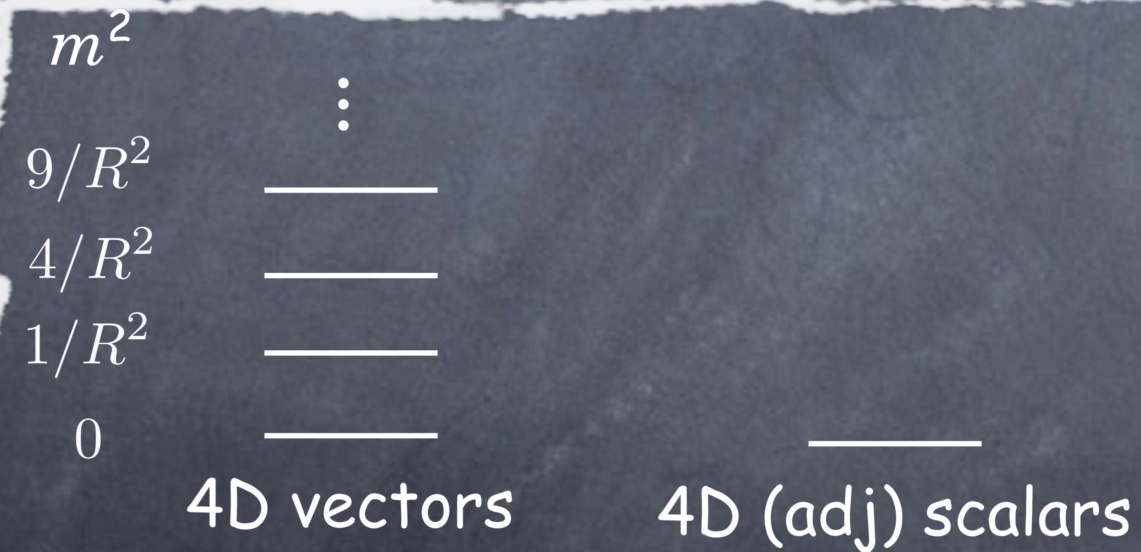
Compactification on a Circle: gauge field

complex notation

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{iny/R} A_M^{(n)}(x) \qquad M = (\mu = 0 \ldots 3, 5)$$

$$\int d^4x\, dy \left(-\frac{1}{4} F_{MN} F^{MN} \right)$$

$$= \int d^4x \left(-\frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\,\mu\nu} + \frac{1}{2} \partial_\mu A_5^{(0)} \partial^\mu A_5^{(0)} + 2 \sum_{n=1}^{\infty} \left(\tilde{F}^{(n)}_{\mu\nu} \tilde{F}^{(-n)\,\mu\nu} + \frac{n^2}{2R^2} \tilde{A}^{(n)}_\mu \tilde{A}^{(-n)\,\mu} \right) \right)$$



massive KK $A_5^{(n)}$ are eaten = longitudinal $A_\mu^{(n)}$

Compactification on a Circle: gauge field

let us turn on some non-trivial non-abelian interactions

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{iny/R} A_M^{(n)}(x)$$

$$D_M = \partial_M + i g_{5D} A_M$$

$[A_M]$=mass$^{3/2}$

$[g_{5D}]$=mass$^{-1/2}$

$$D_\mu = \partial_\mu + i \frac{g_{5D}}{\sqrt{2\pi R}} A_\mu^{(0)} \quad \Rightarrow \quad g_{4D} = \frac{g_{5D}}{\sqrt{2\pi R}}$$

perturbativity holds if

$$g_{4D}^2 \quad > \quad N_{KK} \frac{g_{4D}^4}{16\pi^2}$$

$$N_{KK} < \frac{16\pi^2}{g_{4D}^2}$$

$$\Lambda = \frac{N_{KK}}{R} = \frac{16\pi^2}{g_{4D}^2 R}$$ 5D cutoff

Compactification on a Circle: gauge field

KK unitarization

massive KK gauge boson

non-linear realization of the gauge symmetry

W_L are Goldstone bosons ~ pions of QCD

$$\Sigma = e^{i\sigma^a \pi^a / v} \qquad \mathcal{L}_{\rm mass} = \frac{v^2}{4} \mathrm{Tr}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right)$$

bad behavior of scattering amplitudes

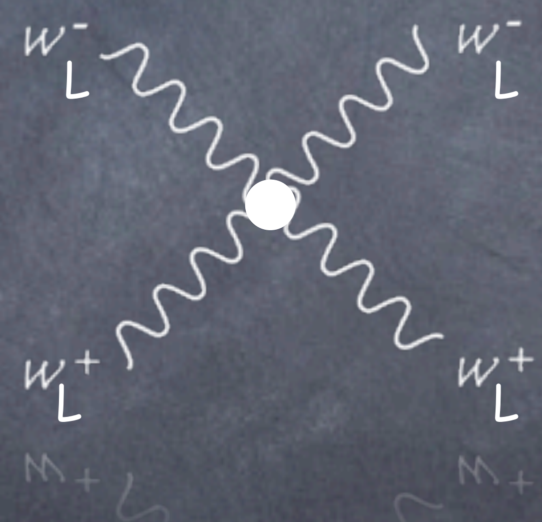
$$\epsilon_l = \left(\frac{|\vec{k}|}{M}, \frac{E}{M}\frac{\vec{k}}{|\vec{k}|} \right)$$

scattering of W_L

~

scattering of QCD pions

(Goldstone equivalence theorem)



$$\mathcal{A} = g^2 \left(\frac{E}{M_W} \right)^2$$

loss of perturbative unitarity

$\Lambda \sim 4\pi M_W/g < \Lambda_{5D}$

the growth of the (elastic) cross section is cancelled by the exchange of KK modes (see Higgsless' lecture)

Compactification on a Circle: fermion

4D Dirac matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$$

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar\sigma^0 = \sigma^0 \qquad \bar\sigma^1 = -\sigma^1 \qquad \bar\sigma^2 = -\sigma^2 \qquad \bar\sigma^3 = -\sigma^3$$

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1_2 & \\ & -1_2 \end{pmatrix}$$

check: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ (+---) metric

5D Dirac matrices

$$\Gamma^\mu = \gamma^\mu \qquad \Gamma^5 = i\gamma^5 = i\begin{pmatrix} 1_2 & \\ & -1_2 \end{pmatrix} \qquad \{\Gamma^M, \Gamma^N\} = 2\eta^{MN}$$

(+----) metric

5D Dirac action

$$\int d^4x dy \left(\frac{i}{2}(\bar\Psi\,\Gamma^M\partial_M\Psi - \partial_M\bar\Psi\,\Gamma^M\Psi) - m\bar\Psi\Psi\right) \qquad \Psi = \begin{pmatrix} \chi \\ \bar\psi \end{pmatrix}$$

5D spinor = 4D Dirac spinor

2 vector-like 2-components spinors

$$= \int d^4x dy \left(-i\bar\chi\bar\sigma^\mu\partial_\mu\chi - i\psi\sigma^\mu\partial_\mu\bar\psi + \tfrac{1}{2}\left(\psi\partial_5\chi - \partial_5\psi\chi - \bar\chi\partial_5\bar\psi + \partial_5\bar\chi\bar\psi\right) + m(\psi\chi + \bar\chi\bar\psi)\right)$$

5D eqs of motion

$$\begin{cases} -i\bar\sigma^\mu\partial_\mu\chi - \partial_5\bar\psi + m\bar\psi = 0 \\ -i\sigma^\mu\partial_\mu\bar\psi + \partial_5\chi + m\chi = 0 \end{cases}$$

Compactification on a Circle: fermion

5D Dirac action

$$\int d^4x dy \left(\frac{i}{2}(\bar\Psi \Gamma^M \partial_M \Psi - \partial_M \bar\Psi \Gamma^M \Psi) - m\bar\Psi\Psi \right)$$

5D eqs of motion

$$\begin{cases} -i\bar\sigma^\mu \partial_\mu \chi - \partial_5 \bar\psi + m\bar\psi = 0 \\ -i\sigma^\mu \partial_\mu \bar\psi + \partial_5 \chi + m\chi = 0 \end{cases}$$

KK decomposition

$$\begin{cases} \chi = \sum_n g_n(y) \chi_n(x) \\ \bar\psi = \sum_n f_n(y) \bar\psi_n(x) \end{cases}$$

$\begin{pmatrix} \chi_n \\ \bar\psi_n \end{pmatrix}$ 4D Dirac spinor of mass m_n

$$\begin{cases} -i\bar\sigma^\mu \partial_\mu \chi_n + m_n \bar\psi_n = 0 \\ -i\sigma^\mu \partial_\mu \bar\psi_n + m_n \chi_n = 0 \end{cases}$$

5D eqs of motion ⇨ diff. eqs for wavefunction

$$g_n' + m\, g_n - m_n f_n = 0$$
$$f_n' - m\, f_n + m_n g_n = 0$$

⇨

$$g_n'' + (m_n^2 - m^2) g_n = 0$$
$$f_n'' + (m_n^2 - m^2) f_n = 0$$

1

$$g_n = \mathcal{N}_n \cos \frac{ny}{R}$$
$$f_n = \mathcal{N}_n \left(\frac{m}{m_n} \cos\frac{ny}{R} - \frac{n}{m_n R} \sin\frac{ny}{R} \right)$$
$$m_n^2 = m^2 + n^2/R^2$$

2

$$g_n = \mathcal{N}_n \sin \frac{ny}{R}$$
$$f_n = \mathcal{N}_n \left(\frac{n}{m_n R} \cos\frac{ny}{R} + \frac{m}{m_n} \sin\frac{ny}{R} \right)$$
$$m_n^2 = m^2 + n^2/R^2$$

remark: there exist zero modes iff m=0

Vector-like spectrum: cannot describe chiral theory as SM

Witten '81

Compactification on a Circle: fermion

KK mode 5D parity

Contrary to scalar/gauge cases, in general KK modes don't have a definite parity y ↔ -y

$$y \to -y \qquad \Psi(y) \to \Gamma^5 \Psi(-y) \qquad \bar\Psi(y) \to \bar\Psi(-y)\Gamma^5$$

- the kinetic term is invariant: $\bar\Psi\Gamma^M\partial_M\Psi \to \bar\Psi(-y)\Gamma^5\left(\Gamma^\mu\partial_\mu - \Gamma^5\partial_{-y}\right)\Gamma^5\Psi(-y)$
$= \bar\Psi(-y)\left(\Gamma^\mu\partial_\mu + \Gamma^5\partial_{-y}\right)\Psi(-y)$
- the mass term is *not* invariant: $\bar\Psi\Psi \to \bar\Psi(-y)\Gamma^5\Gamma^5\Psi(-y) = -\bar\Psi(-y)\Psi(-y)$

definite parity iff m=0, then χ and ψ have opposite parities

KK mode normalization

χ_n and ψ_n have separate kinetic terms ⇨ a priori 2 independent normalization conditions

the 2 normalization conditions are equivalent provided that the quantization eq. holds

$$\mathcal{N}_n = 1/\sqrt{\pi R}$$

$$\int_{-\pi R}^{\pi R} dy\, \cos^2\frac{ny}{R} = \int_{-\pi R}^{\pi R} dy\left(\frac{m}{m_n}\cos\frac{ny}{R} - \frac{n}{m_n R}\sin\frac{ny}{R}\right)^2 = \pi R \qquad \text{iff} \quad m_n^2 = m^2 + n^2/R^2$$

Compactification on a Circle: graviton

massless graviton in D dimensions seen from 4D

$g_{MN}=\eta_{MN}+h_{MN}$ DxD symmetric matrix ⇒ D(D+1)/2 components

D=4 ⇒ 10
D=5 ⇒ 15
D=6 ⇒ 21

- diffeomorphism invariance: $h_{MN}\to h_{MN}+\partial_M\xi_N+\partial_N\xi_M$
  can eliminate D components: e.g., $\partial_M h^{MN}=\frac{1}{2}\partial^N h$ ⇒ D(D-1)/2
- residual invariance: $\Box\xi_N=0$ keeps $\partial_M h^{MN}=\frac{1}{2}\partial^N h$
  can eliminate D more components ⇒ D(D-3)/2

D=4 ⇒ 2
D=5 ⇒ 5
D=6 ⇒ 9

5D

massless level	$g_{\mu\nu}$	$g_{\mu 5}$	g_{55}
5 dof	4D graviton 2	4D vector 2	4D scalar 1
massive level	$g_{\mu\nu}$		
5 dof	4D massive graviton 5		

Compactification on a Circle: graviton

massless graviton in D dimensions seen from 4D

D(D-3)/2 degrees of freedom

(4+n)D			
massless level	$g_{\mu\nu}$	$g_{\mu i}$	g_{ij}
	4D graviton	n 4D vectors	n(n+1)/2 4D scalars
(4+n)(4+n-3)/2 dof	2	2n	n(n+1)/2
massive level	$g_{\mu\nu}$	$g_{\mu i}$	g_{ij}
	1	n-1	n(n-1)/2
	4D massive graviton	4D massive vectors	4D massive scalars
(4+n)(4+n-3)/2 dof	5	3(n-1)	n(n-1)/2

for the explicit KK decomposition of the (4+n)D graviton, see e.g. Giudice, Rattazzi, Wells '98

✓ 1 vector is eaten by the graviton

✓ 1 scalar is eaten by the graviton

✓ (n-1) scalars are eaten by the vectors

Compactification on a Circle: graviton

5D graviton = massless 4D (graviton + vector + scalar)+ massive dof

$$g_{MN} = \eta_{MN} + h_{MN}$$

$$\sqrt{g}\mathcal{R} = \frac{1}{4}\partial_M h \partial^M h - \frac{1}{4}\partial_M h_{NP}\partial^M h^{NP} + \frac{1}{2}\partial_M h^{MP}\partial_N h^N{}_P - \frac{1}{2}\partial_M h^{MN}\partial_N h + \mathcal{O}(h^3)$$

$h_{\mu\nu} = \hat{h}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\phi$ $\qquad$ $h_{\mu 5} = h_{5\mu} = A_\mu$ $\qquad$ $h_{55} = \phi$

$[\hat{h}_{\mu\nu}]$=mass^0 $\qquad$ $[A_\mu]$=mass^0 $\qquad$ $[\phi]$=mass^0

$$\sqrt{g}\mathcal{R} = \sqrt{\hat{g}}\hat{\mathcal{R}} - \frac{1}{8}\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$\int_{-\pi R}^{\pi R} dy\, M_*^3 \sqrt{g}\mathcal{R} = M_{\rm Pl}^2 \sqrt{\hat{g}}\hat{\mathcal{R}} - \frac{1}{2}\sqrt{\hat{g}}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} + \frac{1}{4}\sqrt{\hat{g}}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}$$

$M_{\rm Pl}^2 = 2\pi R M_*^3$ $\qquad$ $\hat{\phi} = \frac{1}{2}M_{\rm Pl}\phi$ $\qquad$ $\hat{A}_\mu = M_{\rm Pl}A_\mu$

the result also holds at the full non-linear level

which legitimates the identification of the Planck scale

(at the quadratic order, one cannot identify the proper noramlization of the graviton)

Compactification on a Circle: graviton

5D graviton = massless dof + massive 4D graviton

As in the previous cases, the derivative along the 5th coordinate gives rise to a mass term
let us look at the Lorentz structure of the KK graviton mass term

$$\partial_5 \to in/R$$

$$\sqrt{g}\mathcal{R} = \tfrac{1}{4}\partial_M h \partial^M h - \tfrac{1}{4}\partial_M h_{NP}\partial^M h^{NP} + \tfrac{1}{2}\partial_M h^{MP}\partial_N h^N{}_P - \tfrac{1}{2}\partial_M h^{MN}\partial_N h + \mathcal{O}(h^3)$$

$$\frac{n^2}{4R^2}h^2 \qquad \frac{n^2}{4R^2}h_{\mu\nu}h^{\mu\nu}$$

non contribution to the mass

Fierz-Pauli structure: $m^2(h^2 - h_{\mu\nu}h^{\mu\nu})$

only structure (in flat space) which doesn't give a ghost/tachyon

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \partial_\mu\partial_\nu\phi$$

ϕ drops out from the graviton kinetic term (gauge invariance) $\sqrt{g}\mathcal{R} = \sqrt{\hat{g}}\hat{\mathcal{R}}$

but appears in the mass term $m^2(\hat{h}^2 + 2\hat{h}\Box\phi + \phi\Box^2\phi - \hat{h}_{\mu\nu}\hat{h}^{\mu\nu} - 2\hat{h}_{\mu\nu}\partial^\mu\partial^\nu\phi - \phi\Box^2\phi)$

the Fierz-Pauli combination is the only one where the four derivative terms cancel out

Compactification on a Circle: graviton

Fierz-Pauli mass term: $m^2(\hat{h}^2 - \hat{h}_{\mu\nu}\hat{h}^{\mu\nu} + 2\hat{h}\Box\phi - 2\hat{h}_{\mu\nu}\partial^\mu\partial^\nu\phi)$

kinetic mixing scalar-graviton

Weyl rescaling to undo the kinetic mixing

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \partial_\mu\partial_\nu\phi = \tilde{h}_{\mu\nu} + \partial_\mu\partial_\nu\phi + 2m^2\eta_{\mu\nu}\,\phi$$

$[\hat{h}_{\mu\nu}]$=mass^0

$[\phi]$=mass^{-2}

$$M_{\rm Pl}^2\left(-\sqrt{g}\mathcal{R} - m^2(h^2 - h_{\mu\nu}h^{\mu\nu})\right) = M_{\rm Pl}^2\left(-\sqrt{\tilde{g}}\tilde{\mathcal{R}} - m^2(\tilde{h}^2 - \tilde{h}_{\mu\nu}\tilde{h}^{\mu\nu}) - 6m^4\phi\Box\phi\right)$$

$$\phi^c = m^2 M_{\rm Pl}\phi$$

canonically normalized

$[\phi^c]$=mass

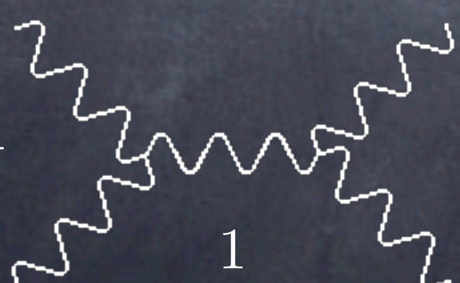
healthy scalar kinetic term

Goldstone self-interactions

$$M_{\rm Pl}^2 m^2(h^2 - h_{\mu\nu}h^{\mu\nu}) = M_{\rm Pl}^2(m^4(\partial\phi)^2 + m^2(\partial^2\phi)^3 + \ldots)$$

$$= (\partial\phi^c)^2 + \frac{1}{m^4 M_{\rm Pl}}(\partial^2\phi^c)^3 + \ldots$$

$\frac{s^3}{m^4 M_{\rm Pl}}$



$\frac{1}{s}$

$\frac{s^3}{m^4 M_{\rm Pl}}$

$$\mathcal{A} \sim \frac{s^5}{m^8 M_{\rm Pl}^2}$$

amplitude becomes strong at $\Lambda \sim \sqrt[5]{m^4 M_{\rm Pl}}$

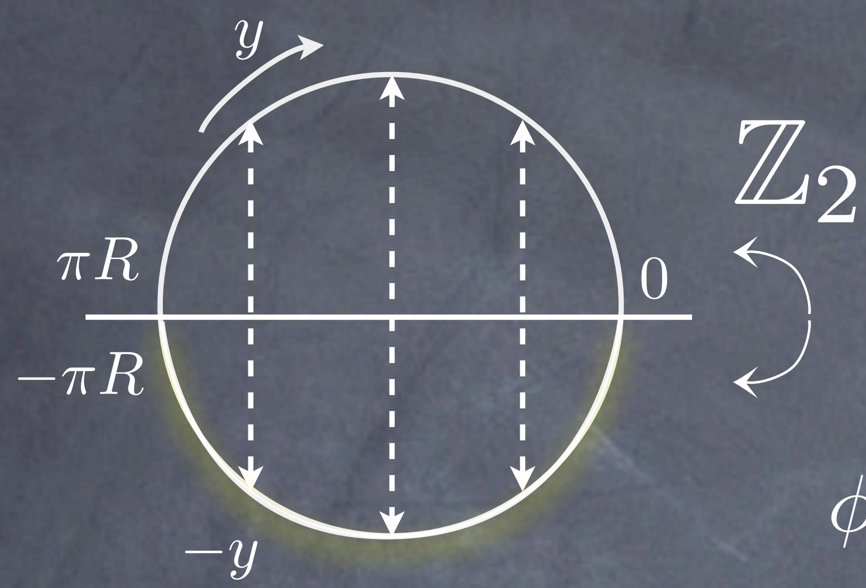
analog of $\Lambda \sim m/g$ in gauge theory

Arkani-Hamed, Georgi, Schwartz '02

(partial) unitarization by KK dynamics? no explicit check!

Orbifold compactification

Compactification on an Orbifold



orbifold = $(\mathbb{R}/T_{2\pi R})/Z_2$

$$y \sim y + 2\pi R \qquad\qquad y \sim -y$$

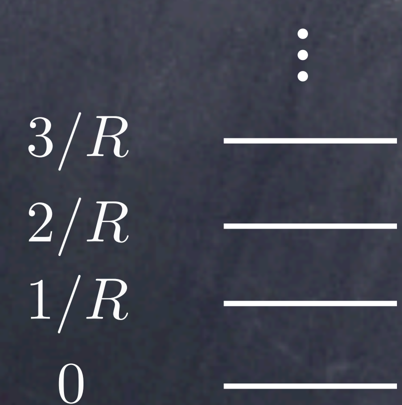
$$\phi(y,x) = \phi(y+2\pi R, x) \qquad \phi(-y,x) = U\phi(y,x)$$

the fields at y and -y
should be equal
up to sym. transformation

$$U^2 = 1$$

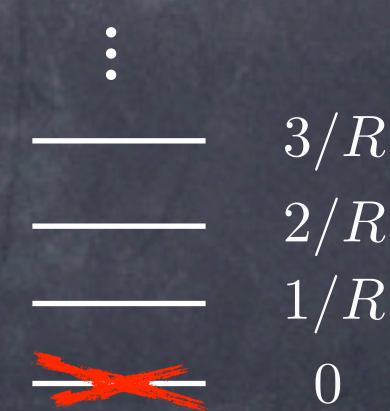
U=+1:

$$\phi(y,x) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n0}}\pi R}} \cos\left(\frac{ny}{R}\right)$$

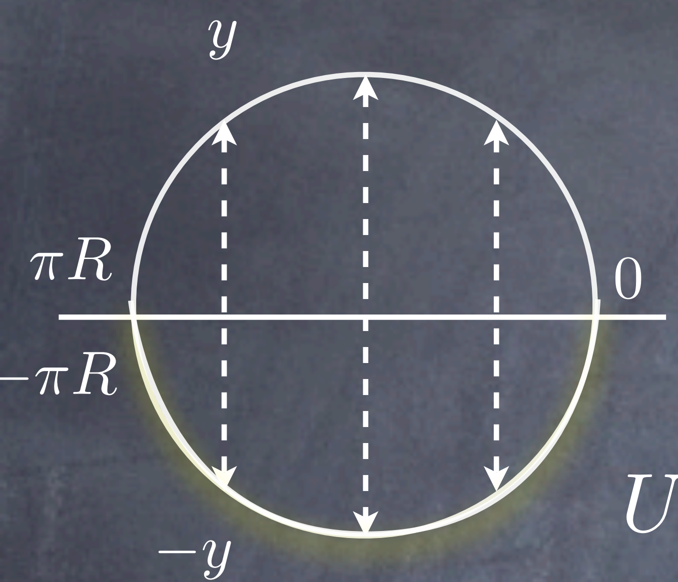


U=-1:

$$\phi(y,x) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \sin\left(\frac{ny}{R}\right)$$



Orbifold Symmetry Breaking



$\mathbb{Z}_2$

$U^2 = 1$

orbifold $y \sim -y$

$$A_\mu(-y) = U A_\mu(y) U^\dagger$$

$$A_5(-y) = -U A_5(y) U^\dagger$$

- signs are compensating

Breaking of gauge group at the end-points of the orbifold $A_\mu(0) = U A_\mu(0) U^\dagger$

at the end-points, the surviving gauge group commute with the orbifold projection matrix U

KK effective theory

zero mode: A_μ is independent of y

$$A_\mu = U A_\mu U^\dagger \qquad A_5 = -U A_5 U^\dagger$$

gauge symmetry breaking

(+ chiral fermions)

SU(3) → SU(2)xU(1) 5D Orbifold Breaking

$$U = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \qquad U \in SU(3) \qquad U^2 = 1$$

massless vectors A_μ

$$[A_\mu, U] = 0 \qquad A_\mu = \frac{1}{2}\begin{pmatrix} A_\mu^3 + A^8/\sqrt{3} & A_\mu^1 - iA_\mu^2 & \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 + A_\mu^8/\sqrt{3} & \\ & & -2A_\mu^8/\sqrt{3} \end{pmatrix} \qquad SU(2)\times U(1)$$

massless scalars A_5

$$\{A_5, U\} = 0 \qquad A_5 = \frac{1}{2}\begin{pmatrix} & & A_5^4 - iA_5^5 \\ & & A_5^6 - iA_5^7 \\ A_5^4 + iA_5^5 & A_5^6 + iA_5^7 & \end{pmatrix} \qquad \frac{SU(3)}{SU(2)\times U(1)}$$

Orbifold Projection as Boundary Conditions

$G \to H$ by orbifold projection

H subgroup

$$A^H_\mu(-y) = A^H_\mu(y) \qquad A^H_5(-y) = -A^H_5(y)$$

which is equivalent to the BCs at the fixed points

$$\partial_5 A^H_\mu = 0 \qquad A^H_5 = 0$$

G/H coset

$$A^{G/H}_\mu(-y) = -A^{G/H}_\mu(y) \qquad A^{G/H}_5(-y) = A^{G/H}_5(y)$$

which is equivalent to the BCs at the fixed points

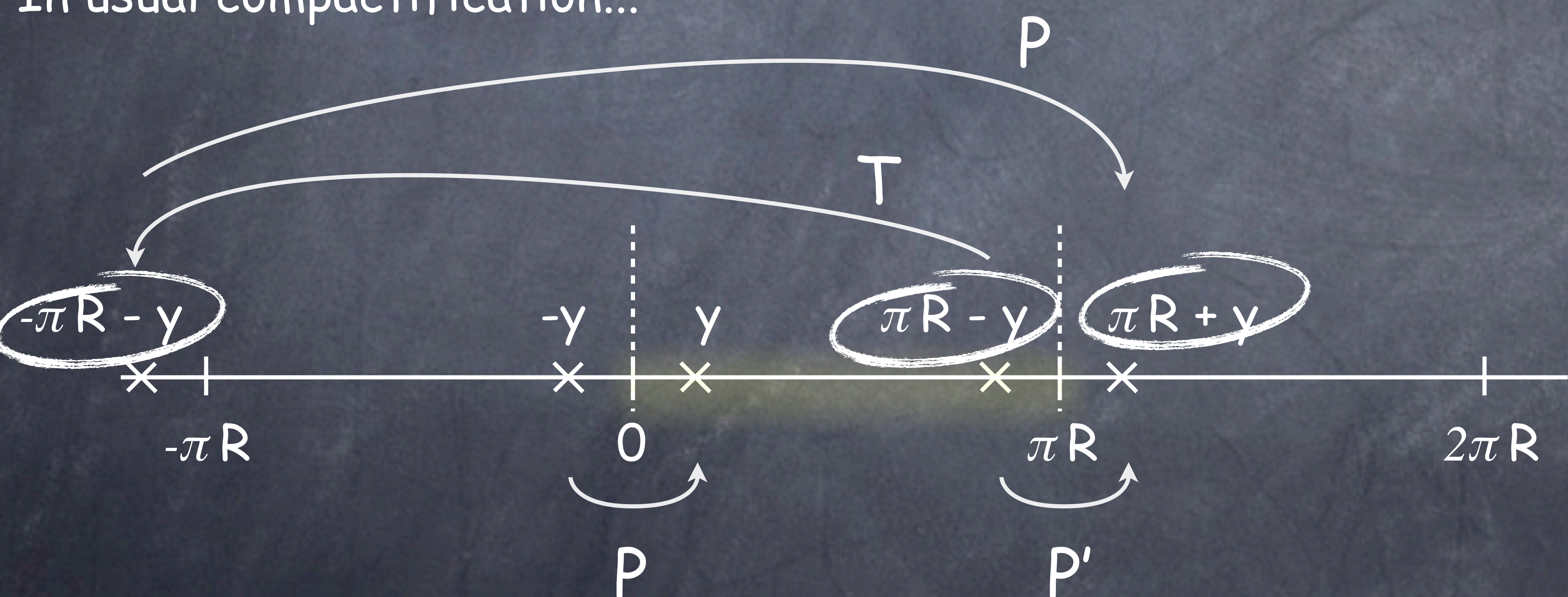
$$A^{G/H}_\mu = 0 \qquad \partial_5 A^{G/H}_5 = 0$$

1D Orbifold

can we have different breaking pattern at the two end-points?

this extra freedom would be needed if we want to reduce the rank of the bulk gauge group

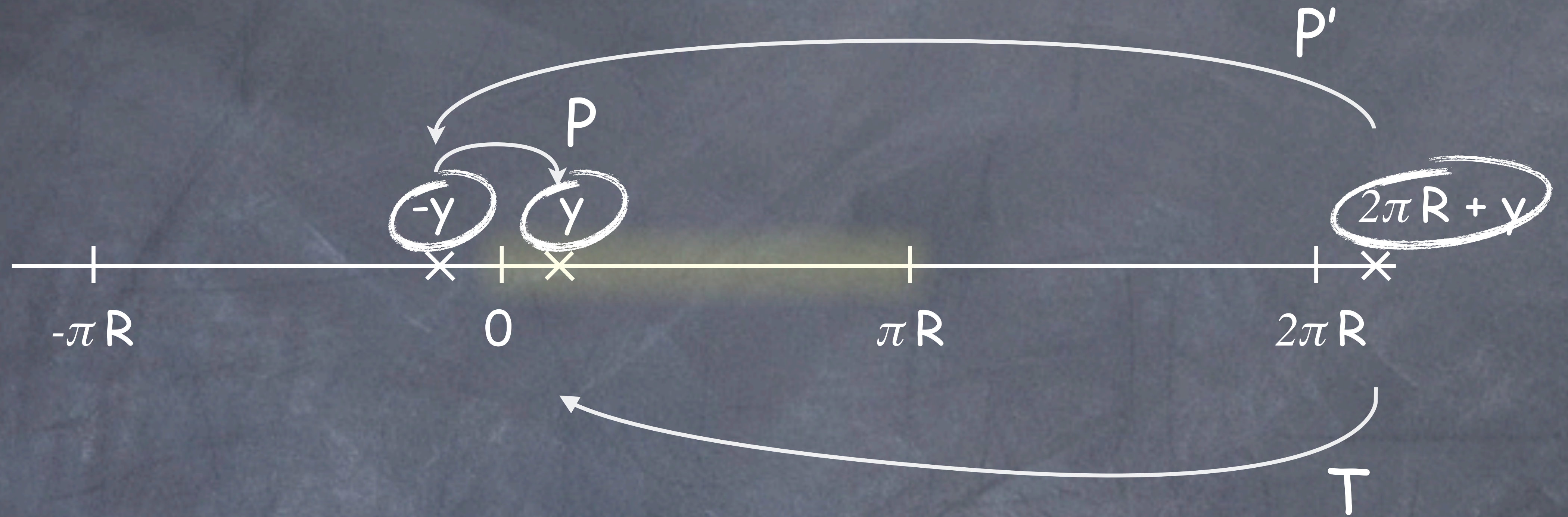
In usual compactification...



P = P'

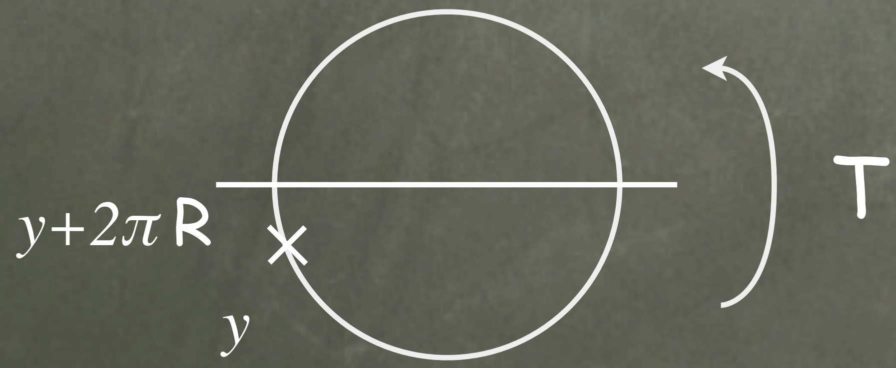
$$\phi(\pi R - y) \underset{T}{=} \phi(-\pi R - y) \underset{P}{=} \mathcal{P}\,\phi(\pi R + y)$$

$Z_2 \times Z'_2$ orbifold



$$\phi(2\pi R + y) = \mathcal{P}'\,\phi(-y) = \mathcal{P}'\mathcal{P}\,\phi(y)$$

$$\mathcal{T} = \mathcal{P}'\mathcal{P}$$



non-trivial compactification
à la Scherk-Schwarz

$$\phi(2\pi R + y) = \mathcal{P}'\mathcal{P}\,\phi(y)$$

Wave-functions for flat space $Z_2 \times Z'_2$ orbifold

assuming Z and Z' commute

KK tower with a massless mode

(+,+) states: $\cos\frac{ny}{R} \Rrightarrow m_n = \frac{n}{R} \quad n = 0 \ldots \infty$

(-,-) states: $\sin\frac{ny}{R} \Rrightarrow m_n = \frac{n}{R} \quad n = 1 \ldots \infty$

(+,-) states: $\cos\frac{(2n+1)y}{2R} \Rrightarrow m_n = \frac{2n+1}{2R} \quad n = 0 \ldots \infty$

(-,+) states: $\sin\frac{(2n+1)y}{2R} \Rrightarrow m_n = \frac{2n+1}{2R} \quad n = 0 \ldots \infty$

Two Examples of Orbifold Breaking

$SU(3)$

$SU(2)\times U(1)$ ——— $SU(2)\times U(1)$

$0 \qquad \pi R$

$(+,+)_\mu$, $(-,-)_5$: $SU(2)\times U(1)$

$(-,-)_\mu$, $(+,+)_5$: $SU(3)/SU(2)\times U(1)$

massless Higgs doublet

$SO(5)\times U(1)_X$

$SU(2)_L\times U(1)_Y$ ——— $SO(4)\times U(1)_X$

$0 \qquad \pi R$

$(+,+)_\mu$, $(-,-)_5$: $SU(2)_L\times U(1)_Y$

$(-,-)_\mu$, $(+,+)_5$: $SO(5)/SO(4)$

massless Higgs doublet

$(-,+)_\mu$, $(+,-)_5$: $\dfrac{SO(4)\times U(1)_X}{SU(2)_L\times U(1)_Y}$

Fermion on Orbifold: Chirality

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}$$

5D spinor = 4D Dirac spinor = 2 vector-like 2-components spinors

flat space...

$$\mathcal{S} = \int d^5x \left(-i\bar\chi\bar\sigma^\mu\partial_\mu\chi - i\psi\sigma^\mu\partial_\mu\bar\psi + \frac{1}{2}\left(\psi\overleftrightarrow{\partial_5}\chi - \bar\chi\overleftrightarrow{\partial_5}\bar\psi\right) + m(\psi\chi + \bar\chi\bar\psi)\right)$$

variation of the action ⇒ bulk eqs. of motion

$$-i\bar\sigma^\mu\partial_\mu\chi - \partial_5\bar\psi + m\bar\psi = 0$$
$$-i\sigma^\mu\partial_\mu\bar\psi + \partial_5\chi + m\chi = 0$$

Boundary conditions:

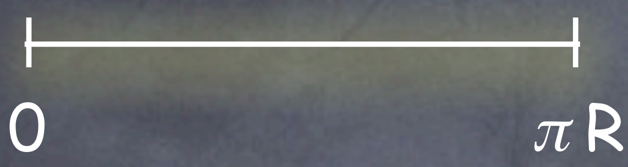
the bulk eqs. evaluated at the boundary couple the two fields:
need to impose BCs only on one field

$$\psi_| = 0 \quad \Leftrightarrow \quad (\partial_5\chi + m\chi)_| = 0$$

different BCs also means chiral spectrum and there should exist a massless mode

massless mode $\psi = 0 \ \ \chi = e^{-my}\tilde\chi(x) \text{ with } -i\bar\sigma^\mu\partial_\mu\tilde\chi = 0$

Fermion on Orbifold: Chirality

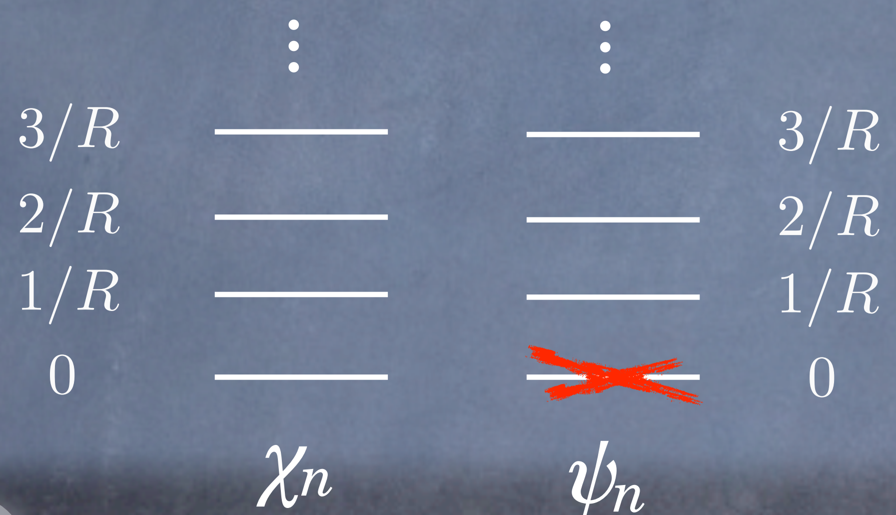


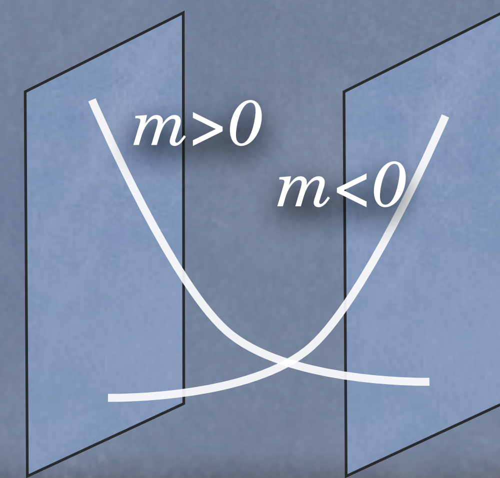
$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}$$

$$-i\bar\sigma^\mu\partial_\mu\chi - \partial_5\bar\psi + m\bar\psi = 0$$
$$-i\sigma^\mu\partial_\mu\bar\psi + \partial_5\chi + m\chi = 0$$

χ zero mode iff ψ is (--) i.e. ψ =0 at y=0 and y=πR

unlike scalar/gauge cases, the zero mode wavefct is not flat in general





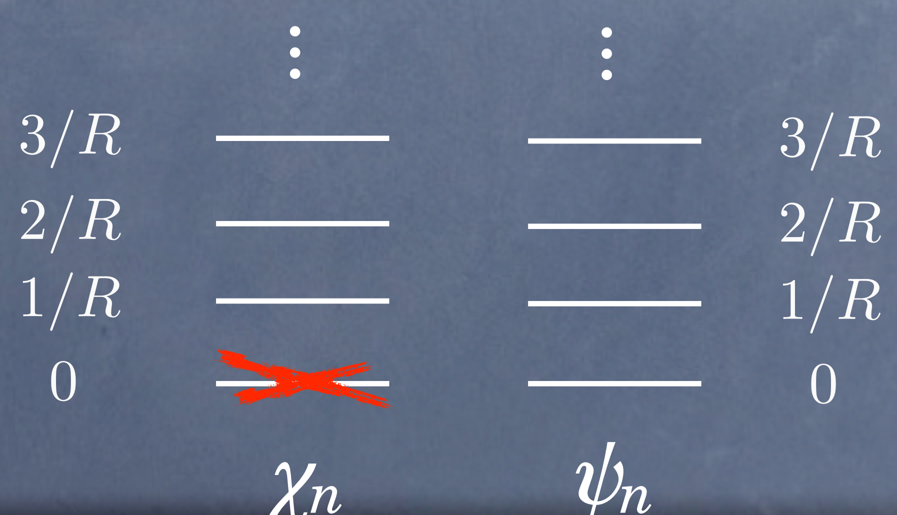
$$\chi_0 = \sqrt{\frac{2m}{1-e^{-2m\pi R}}}\, e^{-my}$$

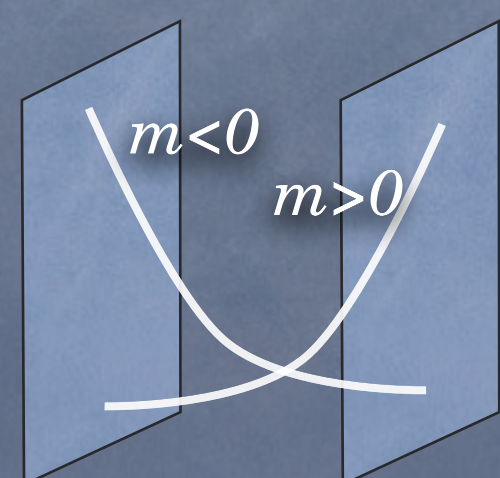
$$\bar\psi_n = \sqrt{\tfrac{2}{\pi R}}\sin\tfrac{ny}{R}$$

$$\chi_n = \sqrt{\tfrac{2}{\pi R}}\left(-\tfrac{n}{m_n R}\cos\tfrac{ny}{R} + \tfrac{m}{m_n}\sin\tfrac{ny}{R}\right)$$

$$m_n^2 = m^2 + n^2/R^2$$

ψ zero mode iff χ is (--) i.e. χ =0 at y=0 and y=πR





$$\bar\psi_0 = \sqrt{\frac{2m}{1-e^{2m\pi R}}}\, e^{my}$$

$$\chi_n = \sqrt{\tfrac{2}{\pi R}}\sin\tfrac{ny}{R}$$

$$\bar\psi_n = \sqrt{\tfrac{2}{\pi R}}\left(\tfrac{n}{m_n R}\cos\tfrac{ny}{R} + \tfrac{m}{m_n}\sin\tfrac{ny}{R}\right)$$

$$m_n^2 = m^2 + n^2/R^2$$

Warped compactification

Anti de Sitter Background

So far we have assumed a flat extra dimension. Let us now move to a curved space

AdS is maximally sym. sol. of Einstein eqs in presence of negative vacuum energy

$$\int d^5x\sqrt{g}\left(-M_5^3\mathcal{R}-\Lambda_5\right) \quad \Rightarrow \quad \mathcal{G}_{MN}\equiv\mathcal{R}_{MN}-\tfrac{1}{2}\mathcal{R}\,g_{MN}=-\tfrac{1}{2M_5^3}\Lambda_5\,g_{MN}$$

Look for a conformally flat solution

$$ds^2=\Omega^2(z)\left(dx_4^2-dz^2\right)$$

$\Omega(z)$ is the "warp" factor

$$\mathcal{G}_{\mu\nu}=-3\frac{\Omega''}{\Omega}\eta_{\mu\nu}=\frac{\Lambda_5}{2M_5^3}\Omega^2\eta_{\mu\nu}$$

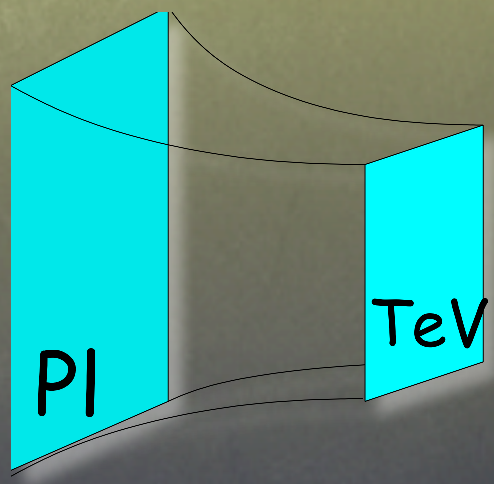
$$\mathcal{G}_{zz}=6\left(\frac{\Omega'}{\Omega}\right)^2=-\frac{\Lambda_5}{2M_5^3}\Omega^2$$

$$\Omega=\frac{R}{z} \qquad R=\sqrt{\frac{12M_5^3}{-\Lambda_5}}$$

Randall-Sundrum background

$$ds^2=\frac{R^2}{z^2}\left(dx_4^2-dz^2\right)$$

$$M_{\rm Pl}^{-1}\sim R<z<R'\sim\ {\rm TeV}^{-1}$$

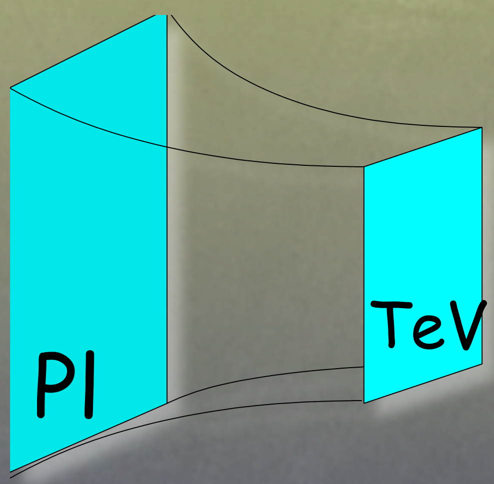


RS solution to the Hierarchy Problem

Randall-Sundrum background

$$ds^2 = \frac{R^2}{z^2}\left(dx_4^2 - dz^2\right)$$

$$M_{\rm Pl}^{-1} \sim R < z < R' \sim \ \text{TeV}^{-1}$$



Higgs on the TeV brane ⇨ its vev gets redshifted to a TeV scale

$$\int d^4x \sqrt{g}\left(g^{\mu\nu}\partial_\mu h \partial_\nu h - \lambda (h^2 - v^2)^2\right)$$

brane localized action
(the Higgs lives on the IR brane)

$$= \int d^4x \left(\frac{R^2}{R'^2}(\partial h)^2 - \lambda \frac{R^4}{R'^4}(h^2 - v^2)^2\right)$$

$$h^c = \frac{R}{R'}\, h$$ is canonically normalized

$$= \int d^4x \left((\partial h^c)^2 - \lambda \left(h^{c2} - \frac{R^2}{R'^2}\, v^2\right)^2\right)$$

effective vev: $v^c = \frac{R}{R'}\, v$ ~ TeV even if v ~ M_{Pl}

Scalars in AdS

AdS $ds^2 = \frac{R^2}{z^2}(dx^2 - dz^2)$ $z = R \dots R'$

$$\mathcal{L} = \int dz \left[\frac{R^3}{z^3} \left(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_z \phi)^2 \right) - \frac{R^5}{z^5}\frac{1}{2} M^2 \phi^2 \right]$$

$$\delta \mathcal{L} = 0 \quad \Rightarrow \quad \underbrace{-\partial_\mu^2}_{m^2} \phi + \frac{z^3}{R^3} \partial_z \left(\frac{R^3}{z^3} \partial_z \phi \right) - \frac{R^2}{z^2} M^2 \phi = 0$$

exercise

$$\phi_n(z) = \frac{z^2}{\mathcal{N}_n^2} (J_\nu(m_n z) + b_n Y_\nu(m_n z))$$

$$\nu^2 = 4 + M^2 R^2$$

Scalars in AdS $Z_2 x Z'_2$ Orbifold

assuming Z and Z' commute

$$\phi_n(z) = \frac{z^2}{\mathcal{N}_n^2}(J_\nu(m_n z) + b_n Y_\nu(m_n z))$$

- (-,-) states: $\phi_{|z=R,R'} = 0 \quad \Rrightarrow \quad \frac{J_\nu(mR)}{Y_\nu(mR)} = \frac{J_\nu(mR')}{Y_\nu(mR')}$

  discrete spectrum $\quad m_n \sim (n + \nu/2 - 1/4)\pi/R'$

- (+,+) states: $(\partial_z \phi)_{|z=R,R'} = 0$

$$\Rrightarrow \quad \frac{(2-\nu)J_\nu(mR) + mRJ_{\nu-1}(mR)}{(2-\nu)Y_\nu(mR) + mRY_{\nu-1}(mR)} = \frac{(2-\nu)J_\nu(mR') + mR'J_{\nu-1}(mR')}{(2-\nu)Y_\nu(mR') + mR'Y_{\nu-1}(mR')}$$

  discrete spectrum $\quad m_n \sim (n + \nu/2 - 3/4)\pi/R'$

Gauge Fields in AdS

AdS $\quad ds^2 = \frac{R^2}{z^2}(dx^2 - dz^2) \qquad z = R \ldots R'$

$$\mathcal{L} = -\frac{1}{4g_5^2}\int dz F_{MN}F^{MN} = -\frac{1}{4g_5^2}\int dz \frac{R}{z}\left((\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - 2(\partial_z A_\mu)^2\right)$$

$$\delta\mathcal{L} = 0 \quad \Rrightarrow \quad \partial^\nu(\partial_\nu A_\mu - \partial_\mu A_\nu) - \frac{z}{R}\partial_z\left(\frac{R}{z}\partial_z A_\mu\right) = 0$$

exercise

$$A_\mu^{(n)}(z) = \frac{z}{\mathcal{N}_n^2}(J_1(m_n z) + b_n Y_1(m_n z))$$

- (+,+) states: $\quad \partial_z A_\mu^{(n)}{}_{|z=R,R'} = 0 \Rrightarrow \frac{J_0(mR)}{Y_0(mR)} = \frac{J_0(mR')}{Y_0(mR')}$

$$mR' \sim 2.44,\ 5.56,\ 8.70,\ 11.83\ldots$$

Fermions in AdS: Partial Compositeness

Grossman and Neubert, '00
Gherghetta and Pomarol, '00

$$\mathcal{S} = \int d^5x \frac{R^4}{z^4} \left(-i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\psi\sigma^\mu\partial_\mu\bar{\psi} + \tfrac{1}{2}\left(\psi \overleftrightarrow{\partial_5} \chi - \bar{\chi} \overleftrightarrow{\partial_5} \bar{\psi}\right) + \frac{c}{z}(\psi\chi + \bar{\chi}\bar{\psi}) \right)$$

5D mass term in AdS unit: c ~ O(1)

bulk eqs of motion:

$$-i\bar{\sigma}^\mu\partial_\mu\chi - \partial_5\bar{\psi} + \frac{c+2}{z}\bar{\psi} = 0$$

$$-i\sigma^\mu\partial_\mu\bar{\psi} + \partial_5\chi + \frac{c-2}{z}\chi = 0$$

wavefunctions

$$\chi = (mz)^{5/2}\left(a_n J_{1/2+c}(mz) + b_n J_{-1/2-c}(mz)\right)$$

$$\psi = (mz)^{5/2}\left(a_n J_{-1/2+c}(mz) - b_n J_{1/2-c}(mz)\right)$$

fermion zero mode:

$$\chi = a_0 \left(\frac{z}{z_{UV}}\right)^{2-c} \tilde{\chi}_{4D} \quad \text{with} \quad \int_{z_{IR}}^{z_{UV}} dz\, a_0^2 \left(\frac{z}{z_{UV}}\right)^{2-c} = 1$$

$c > 1/2$: the zero is normalizable when z^{IR} is sent to infinity (no IR brane): UV localized

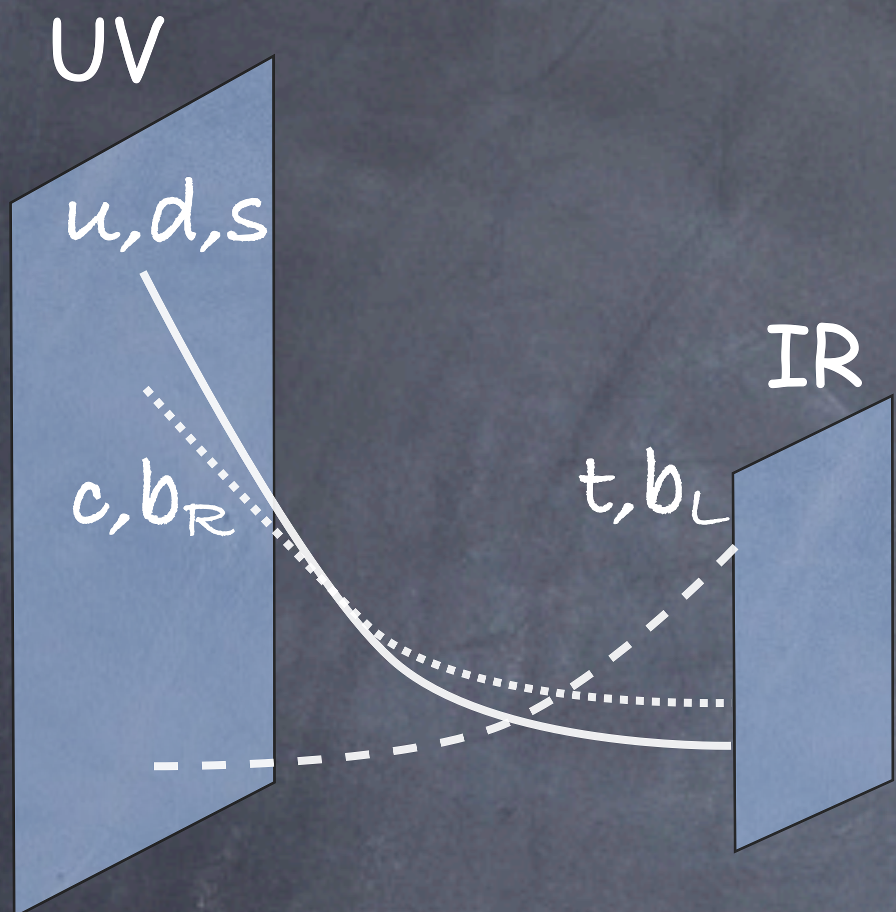
Elementary fermion

$c < 1/2$: the zero is normalizable when z^{UV} is sent to 0 (no UV brane): IR localized

Composite fermion

Masses from IR overlaps

[Grossman and Neubert, '00]
[Gherghetta and Pomarol, '00]
[Huber, '03]



fermion zero-mode has an exponential profile in the bulk

$$\chi(z) = \frac{f_c}{\sqrt{R'}}\left(\frac{z}{R}\right)^2\left(\frac{z}{R'}\right)^{-c}$$

f_c is the "value" of wavefct. on the IR:

$$f_c = \sqrt{\frac{1-2c}{1-(R/R')^{1-2c}}}$$

c < 1/2: heavy fermion $f_c \sim \mathcal{O}(1)$

c > 1/2: light fermion $f_c \sim (R/R')^{c-1/2} \ll 1$

light fermion exponentially localized on the UV brane

➲ overlap with Higgs vev on the IR tiny

➲ exponentially small 4D mass

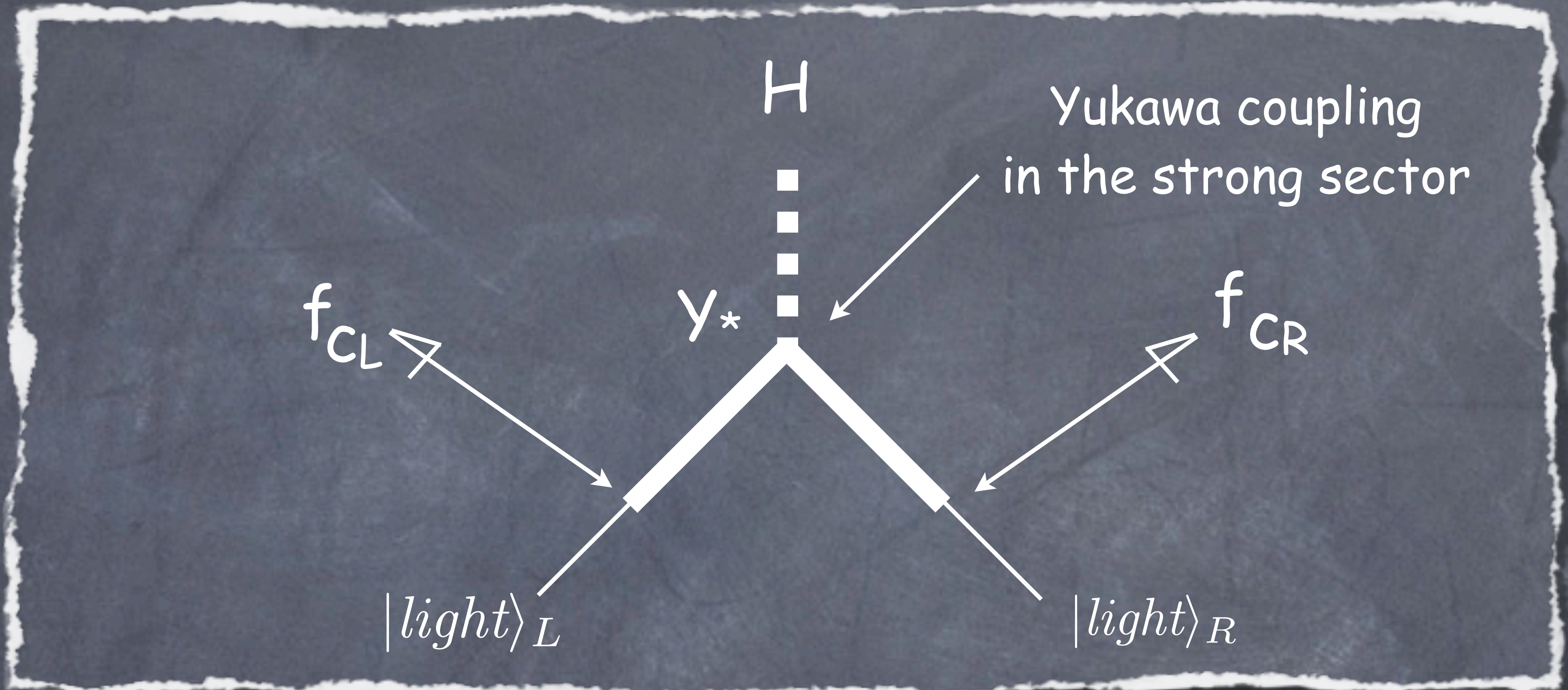
partial compositeness

zero is mixture of elementary and composite fermion

f_c is the amount of compositness

Partial Compositeness: Yukawa Couplings

Higgs part of the strong sector: it couples only to composite fermions



when the Higgs gets a vev, the light dof will acquire a mass prop. to

$$Y^{eff} = Y_\star\, f_{c_L} f_{c_R}$$

Yukawa hierarchy comes from the hierarchy of compositeness

~ the 5D picture gives a rationale for hierarchical f_c ~

Anarchy: mixing angles from mass hierarchy

[Froggatt, Nielsen '79]

$$Y^{eff}_{d_{ij}} = Y^{\star}_{d_{ij}} f_{q_i} f_{d_j} \qquad Y^{eff}_{u_{ij}} = Y^{\star}_{d_{ij}} f_{q_i} f_{u_j}$$

Y_u, Y_d ~O(1): anarchic structure

f_i: hierarchic structure: $f_1 \ll f_2 \ll f_3$

Not only, it leads to a hierarchical spectrum

$$m_{u_i} \propto f_{q_i} f_{u_i} \qquad m_{d_i} \propto f_{q_i} f_{d_i}$$

It also gives hierarchical angles

$$U_{uL} Y^{eff}_u U^\dagger_{uR} = \text{diag} \qquad U_{dL} Y^{eff}_d U^\dagger_{dR} = \text{diag}$$

with (for i < j)

$$U^{ij}_{uL,dL} \sim f_{q_i}/f_{q_j} \qquad U^{ij}_{uR} \sim f_{u_i}/f_{u_j} \qquad U^{ij}_{dR} \sim f_{d_i}/f_{d_j}$$

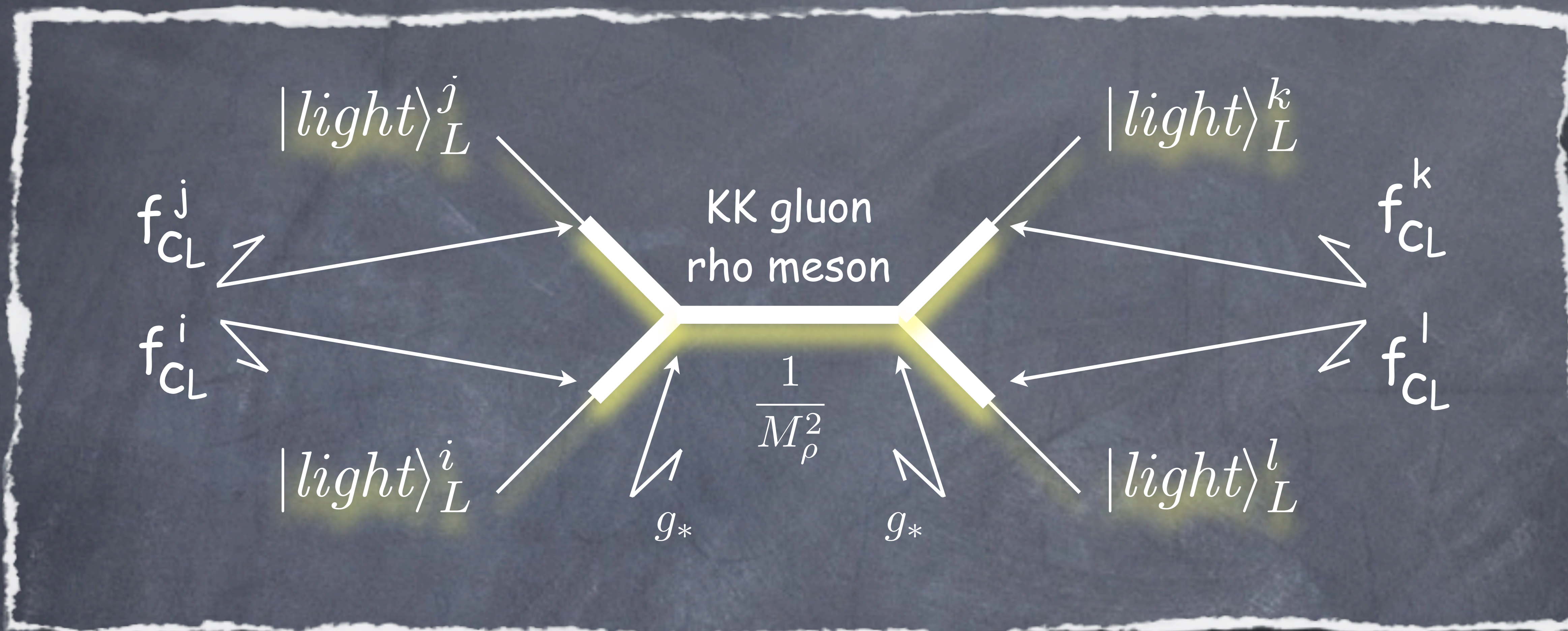
and therefore, we also get

$$V^{ij}_{CKM} \sim f_{q_i}/f_{q_j}$$

➲ alignment angles/masses nicely explained ⊂

FCNC from KK gluons/rho meson

Agashe, Perez, Soni '04
Contino, Kramer, Son, Sundrum '06



$$\mathcal{A}^{ijkl}_{LL} \sim \frac{g_\star^2}{m_\rho^2} f_{c_L^i} f_{c_L^j} f_{c_L^k} f_{c_L^l}$$

Built-in GIM supression

smaller the mass ⊃ smaller the compositeness ⊃ smaller the amplitude

structure similar to the general set-up recently proposed by Davidson et al.

"Solving the flavour problem with hierarchical fermion wave functions", 0711.3376

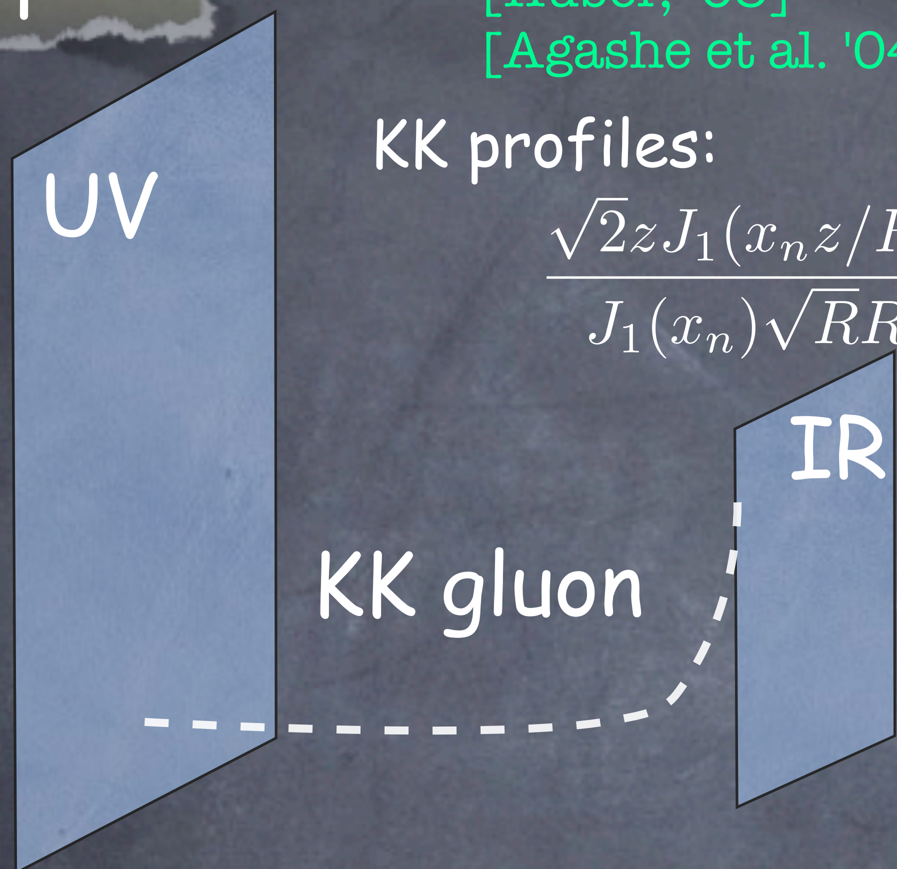
RS-GIM suppression of FCNC

warped

[Gherghetta, Pomarol '00]
[Huber, '03]
[Agashe et al. '04]

KK profiles:

$$\frac{\sqrt{2} z J_1(x_n z/R')}{J_1(x_n)\sqrt{R}R'}$$



KK gluons are flat in UV ⊃ flavor universal
flavor violation are coming from IR
FCNC are suppressed for light fermions

$$g_{\tilde{g}_{KK} Q_L^i Q_L^i} \sim g_\star \left(-\frac{\mathcal{O}(1)}{\log R'/R} + \mathcal{O}(1) f^2_{c_L^i} \right)$$

$$\Downarrow$$

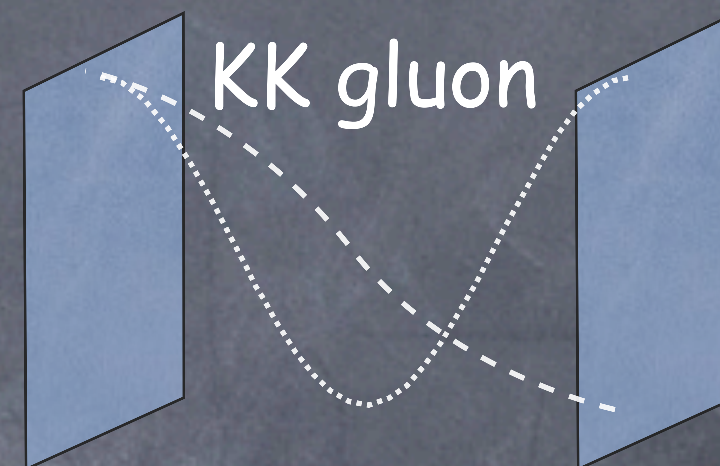
$$g_{\tilde{g}_{KK} Q_L^i Q_L^j} \propto g_\star f_{c_L^i} f_{c_L^j}$$

"low" KK scale allowed

flat

KK profiles:

$$\sqrt{\frac{2}{L}} \cos(n\pi z/L)$$



KK gluons are spread along the extra-dim.
feel all differences in fermion profiles
maximal flavour violation

"high" KK scale required

Extra Dimensions for TeV Physics

Parma International School of Theoretical Physics
"Theoretical Tools for the LHC"
Parma, August 31-September 4, 2009

Christophe Grojean
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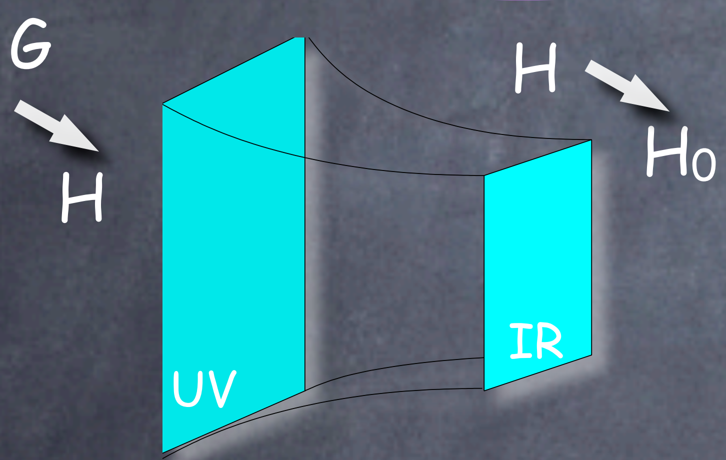
Holographic Approach to Strong Sector

"AdS/CFT" correspondence for model-builder

Warped gravity with fermions and gauge field in the bulk and Higgs on the brane ⟷ Strongly coupled theory with slowly-running couplings in 4D



AdS = warped space

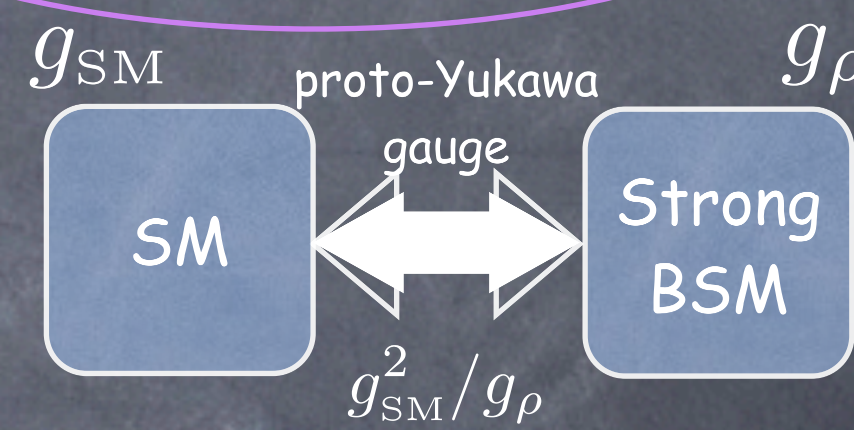
curvature ~ $1/M_{Pl}$

size ~ $40/M_{Pl}$

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

exponential red-shift

$$\frac{R_{UV}}{R_{IR}} \sim 10^{-16}$$



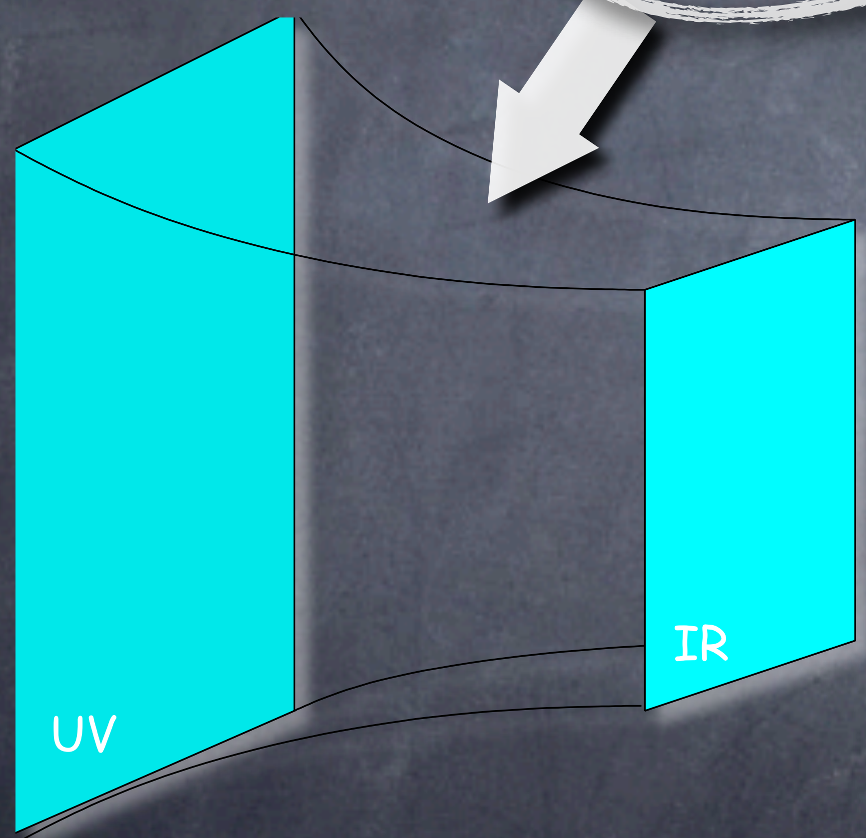
5D	4D
KK modes	vector resonances (ρ mesons in QCD)
motion along 5th dim	RG flow
UV brane	UV cutoff
IR brane	break. of conformal inv.
bulk local sym.	global sym.

Holographic Models of EWSB

Original Randall-Sundrum proposal: '99

Gravity in the bulk

All SM fields on the IR brane



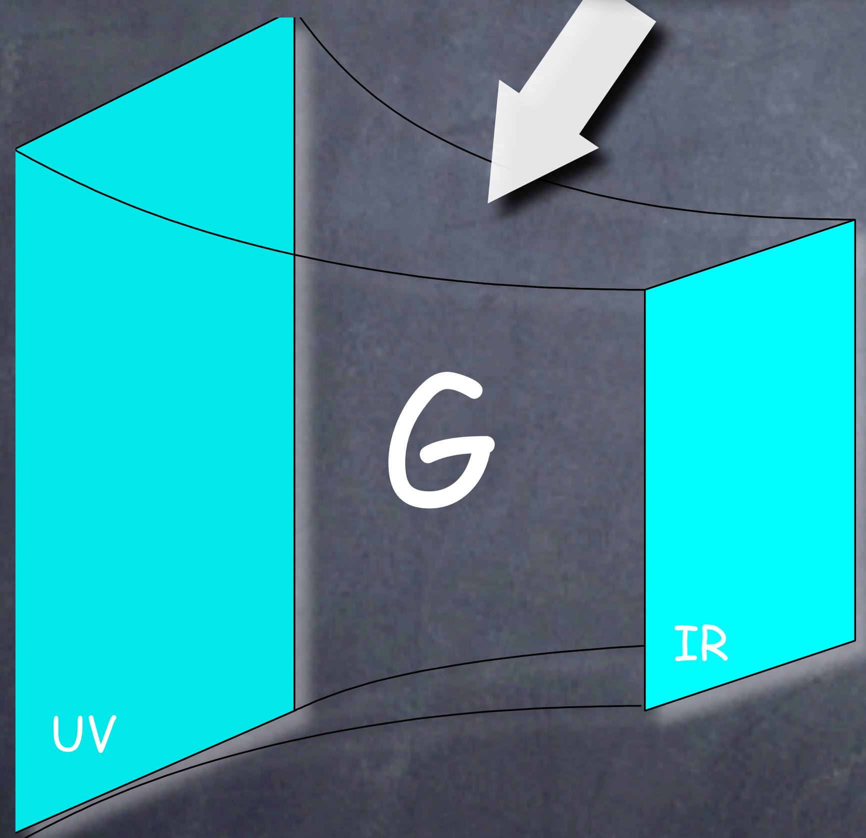
- cutoff ~ 1 TeV
- conflict with EW precision data
- problems with flavour

Holographic Models of EWSB

Bulk gauge fields: Pomarol, '00
Holographic technicolor=Higgsless: Csaki et al., '03
Holographic composite Higgs: Agashe et al., '04

Gauge fields + fermions in the bulk

Higgs on the IR brane or Gauge breaking by boundary conditions



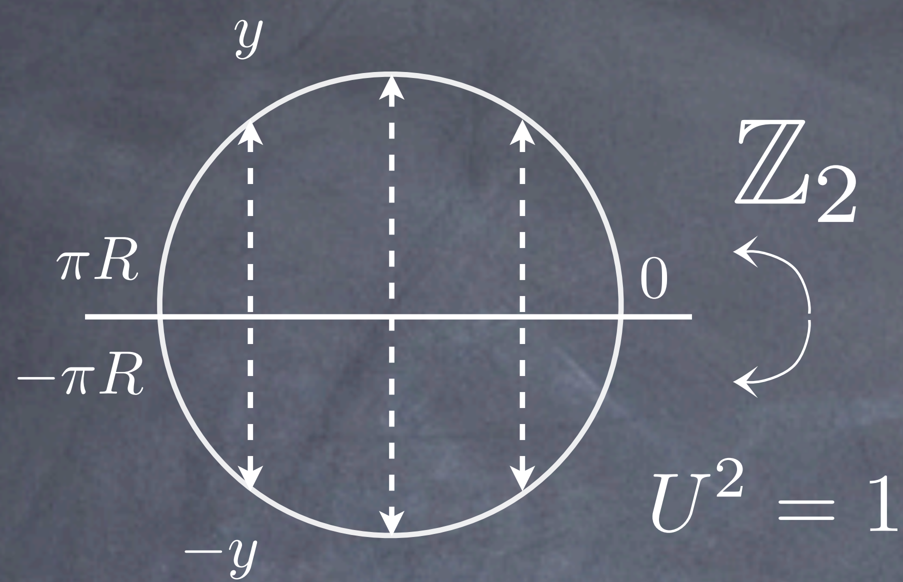
$G=SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$G=SO(5) \times U(1)_X$
$G=SO(6) \times U(1)_X$

- UV completion: log running of gauge couplings
- Custodial symmetry from bulk $SU(2)_R$
- Dynamical 'explanation' of fermion masses
- Built-in flavour structure

Higgsless Models

Higgsless Approach

Csaki, Grojean, Murayama, Pilo, Terning '03
Csaki, Grojean, Pilo, Terning '03



Gauge symmetry breaking

In orbifold compactification, we have seen that we can break gauge symmetry by appropriate boundary conditions

Why can't we break directly SU(2)xU(1) to $U(1)_{em}$ by orbifold?

Dynamical Origin of the BCs

$$\mathcal{S} = \int d^4x \int_0^{\pi R} dy \left(\frac{1}{2} \underbrace{\partial_M \phi \partial^M \phi} - V(\phi) \right) - \int_{y=0,\pi R} d^4x \, \frac{1}{2} M^2_{0,\pi R} \phi^2$$

integration by part

$$\delta S = \int_{y=0,\pi R} d^4x \, \delta\phi \left(\partial_5 \phi + M^2_{0,\pi R} \phi \right)$$

BC's

$$\delta\phi \left(\partial_5 \phi + M^2_{0,\pi R} \phi \right) = 0$$

+

Bulk Part

bulk eq. of motion

$$\Box_5 \phi = -V'(\phi)$$

Dirichlet BC: $\phi = \text{cst.}$

Mixed BC: $\partial_5 \phi_{0,\pi R} = -M^2_{0,\pi R} \phi_{0,\pi R}$

$M^2 \to \infty$: $\phi_{0,\pi R} = 0$ Dirichlet BC

$M^2 \to 0$: $\partial_5 \phi_{0,\pi R} = 0$ Neumann BC

Higgsless Models

mass without a Higgs

$$m^2 = E^2 - \vec{p}_3{}^2 - \vec{p}_\perp{}^2$$

momentum along extra dimensions ~ 4D mass

quantum mechanics in a box

⇓

boundary conditions generate a transverse momentum

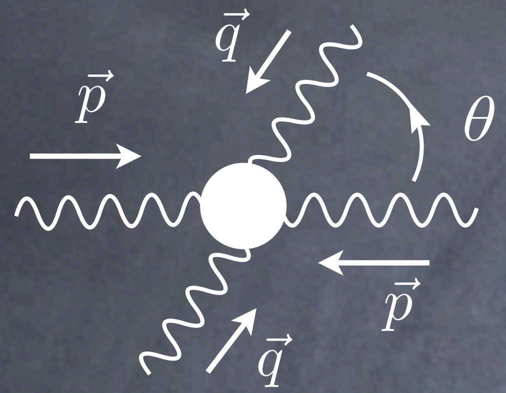
Is it better to generate a transverse momentum than introducing by hand a symmetry breaking mass for the gauge fields?

ie how is unitarity restored without a Higgs field?

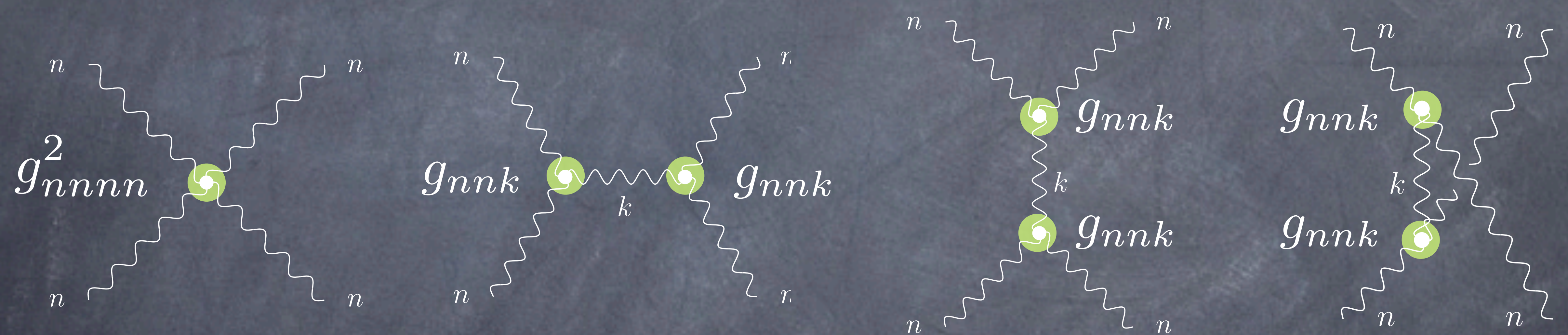
Unitarization of (Elastic) Scattering Amplitude

Same KK mode
'in' and 'out'

$$\epsilon_\perp^\mu = \left(\frac{|\vec{p}|}{M}, \frac{E\ \vec{p}}{M\ |\vec{p}|}\right)$$



$$\mathcal{A} = \mathcal{A}^{(4)}\left(\frac{E}{M}\right)^4 + \mathcal{A}^{(2)}\left(\frac{E}{M}\right)^2 + \dots$$



contact interaction

s channel exchange

t channel exchange

u channel exchange

$$\mathcal{A}^{(4)} = i\left(g^2_{nnnn} - \sum_k g^2_{nnk}\right)\left(f^{abe}f^{cde}(3+6c_\theta - c_\theta^2) + 2(3-c_\theta^2)f^{ace}f^{bde}\right)$$

$$\mathcal{A}^{(2)} = i\left(4g^2_{nnnn} - 3\sum_k g^2_{nnk}\frac{M_k^2}{M_n^2}\right)\left(f^{ace}f^{bde} - s^2_{\theta/2}f^{abe}f^{cde}\right)$$

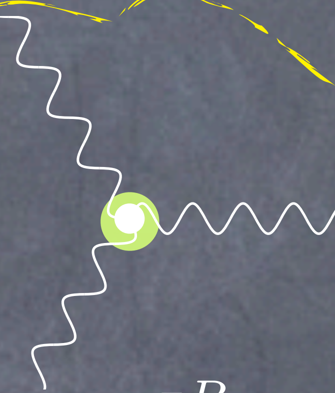
KK Sum Rules

Csaki, Grojean, Murayama, Pilo, Terning '03

$$\mathcal{A}^{(4)} \propto g_{nnnn}^2 - \sum_k g_{nnk}^2 \qquad \mathcal{A}^{(2)} \propto 4g_{nnnn}^2 - 3\sum_k g_{nnk}^2 \frac{M_k^2}{M_n^2}$$

In a KK theory, the effective couplings are given by overlap integrals of the wavefunctions

$$g_{mnpq}^2 = g_{5D}^2 \int_0^{\pi R} dy f_m(y) f_n(y) f_p(y) f_q(y)$$



$$g_{mnp} = g_{5D} \int_0^{\pi R} dy f_m(y) f_n(y) f_p(y)$$

E^4 Sum Rule

$$g_{nnnn}^2 - \sum_k g_{nnk}^2 = g_{5D}^2 \int_0^{\pi R} dy f_n^4(y) - g_{5D}^2 \int_0^{\pi R} dy \int_0^{\pi R} dz f_n^2(y) f_n^2(z) \sum_k f_k(y) f_k(z) = 0$$

$$\sum_k f_k(y) f_k(z) = \delta(y-z)$$

Completness of KK modes

$$\mathcal{A}^{(4)} = 0$$

Postponing Pert. Unitarity Breakdown

Is it a counter-example of the theorem by Cornwall et al.?

i.e. can we unitarize the theory without scalar field?

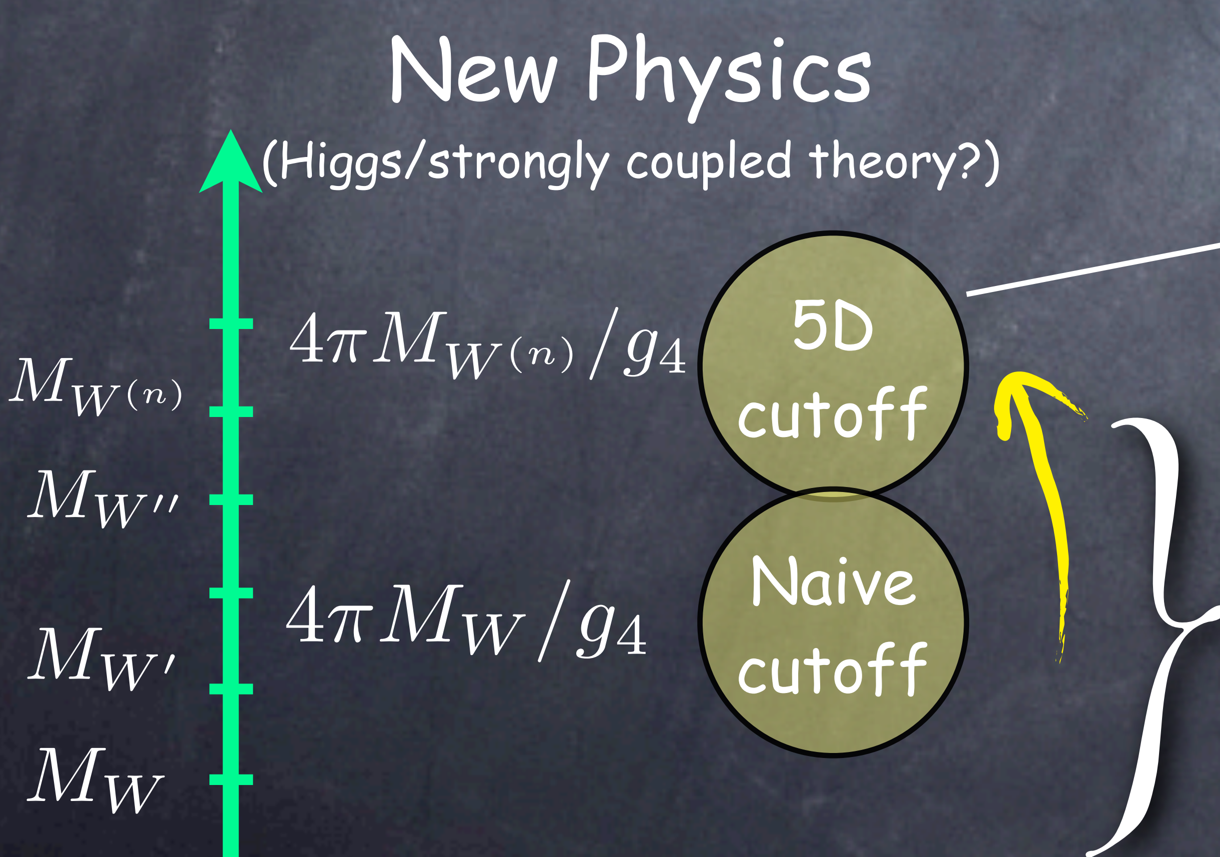
No!

$$g^2_{nnnn} \overset{E^4}{=} \sum_k g^2_{nnk} \overset{E^2}{=} \sum g^2_{nnk} \frac{3M_k^2}{4M_n^2}$$

the sum rules cannot be satisfied with a finite number of KK modes

(to unitarize the scattering of massive KK modes, you always need heavier KK states)

Pushing the need for a scalar to higher scale



With a finite number of KK modes

not directly set by the weak scale

flat space

$$\Lambda_{5D} = 24\pi^3/g_5^2 = (3\pi/g_4)\,\Lambda_{4D}$$

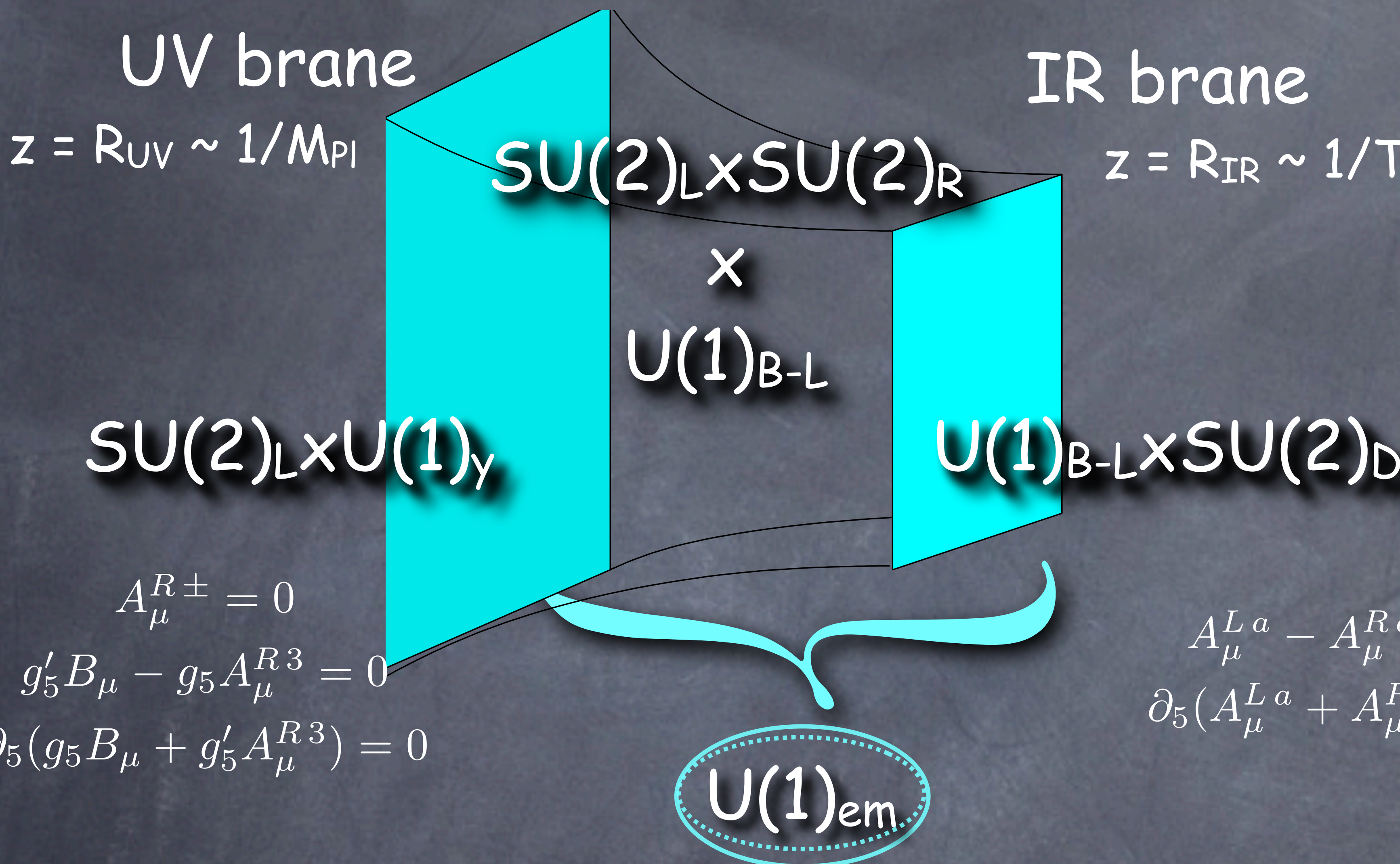
$$\left(g_4 = g_5/\sqrt{2\pi R} \ \ \& \ \ M_W = 1/R \right)$$

a factor 15 higher than the naive 4D cutoff

thanks to the non-trivial KK dynamics

Warped Higgsless Model

Csaki, Grojean, Pilo, Terning '03



$$A_\mu^{R\,\pm} = 0$$
$$g_5' B_\mu - g_5 A_\mu^{R\,3} = 0$$
$$\partial_5 (g_5 B_\mu + g_5' A_\mu^{R\,3}) = 0$$

$$A_\mu^{L\,a} - A_\mu^{R\,a} = 0$$
$$\partial_5 (A_\mu^{L\,a} + A_\mu^{R\,a}) = 0$$

$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$\Omega = \frac{R_{IR}}{R_{UV}} \approx 10^{16}\ \text{GeV}$$

BCs kill all A_5 massless modes: no 4D scalar mode in the spectrum

"light" mode:

$$M_W^2 = \frac{1}{R_{IR}^2 \log(R_{IR}/R_{UV})}$$

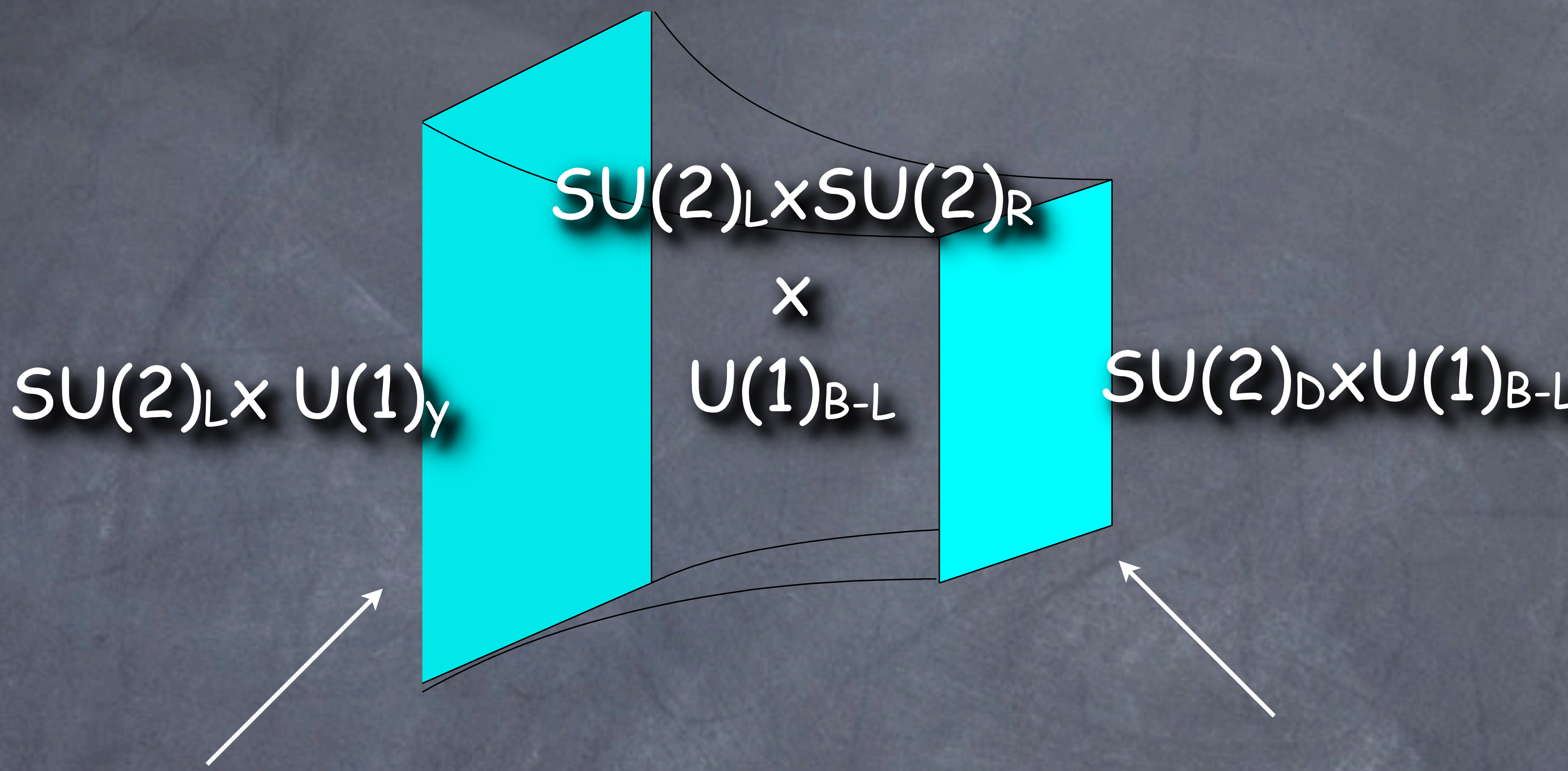
$$M_Z^2 \sim \frac{g_5^2 + 2g_5'^2}{g_5^2 + g'^2} \frac{1}{R_{IR}^2 \log(R_{IR}/R_{UV})}$$

log suppression

KK tower:

$$M_{KK}^2 = \frac{\text{cst of order unity}}{R_{IR}^2}$$

SM Fermions in Higgsless Models



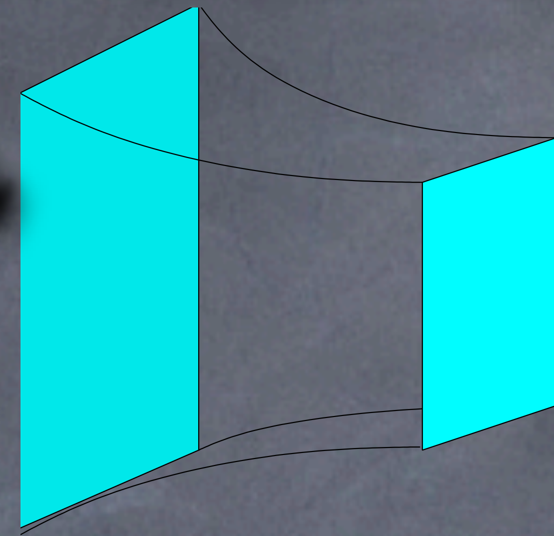
Chiral brane
no possible mass term

vector-like brane
isospin invariant mass only
(same mass for the top and bottom or electron and neutrino)

The fermions have to live in the bulk

Fermion Masses

$SU(2)_L \times U(1)_Y$



$SU(2)_D \times U(1)_{B-L}$

isospin splitting

$-i\kappa\psi_{d_R}\sigma^\mu\partial_\mu\bar{\psi}_{d_R}$

vector-like mass

$R_{IR}M_D\left(\chi_{u_L}\psi_{u_R}+\chi_{d_L}\psi_{d_R}+h.c.\right)$

brane operators will modify the BCs

- Vector like mass $\qquad M_D$

$\chi_L \; +$
$\psi_L \; -$
$\chi_R \; -$
$\psi_R \; +$

$\psi_{L|\text{TeV}}=0$

$\chi_{R|\text{TeV}}=0$

discontinuities in χ_L & ψ_R

$\psi_{L|\text{TeV}}=-M_D R_{IR}\,\psi_{R|\text{TeV}}$

$\chi_{R|\text{TeV}}=M_D R_{IR}\,\chi_{L|\text{TeV}}$

- Isospin splitting $\qquad \kappa$

$\chi_{u_R} \; -$
$\psi_{u_R} \; +$

$\chi_{u_R|\text{UV}}=0$

discontinuities in ψ_{u_R}

$\chi_{u_R|\text{UV}}=\kappa m\psi_{u_R|\text{UV}}$

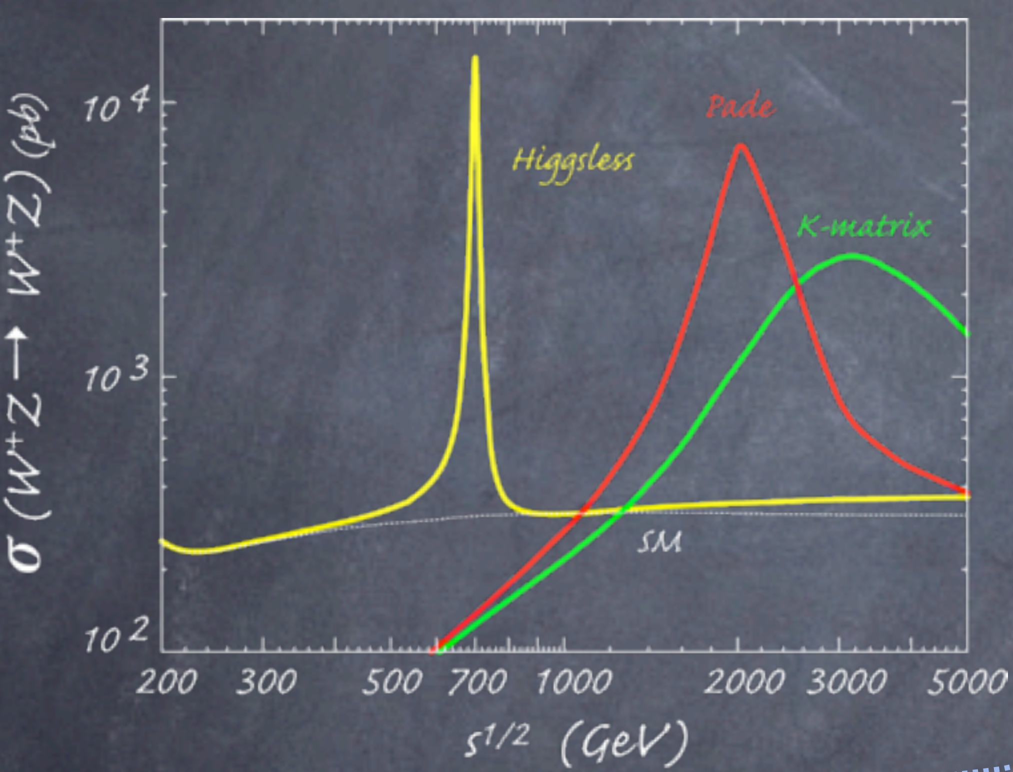
$$m\approx\frac{\sqrt{2c_L-1}}{\sqrt{\kappa^2-1/(2c_R+1)}}M_D\left(\frac{R_{UV}}{R_{IR}}\right)^{c_L-c_R-1}$$

Collider Signatures

Birkedal, Matchev, Perelstein '05
He et al. '07

unitarity restored by vector resonances whose masses and couplings are constrained by the unitarity sum rules

WZ elastic cross section

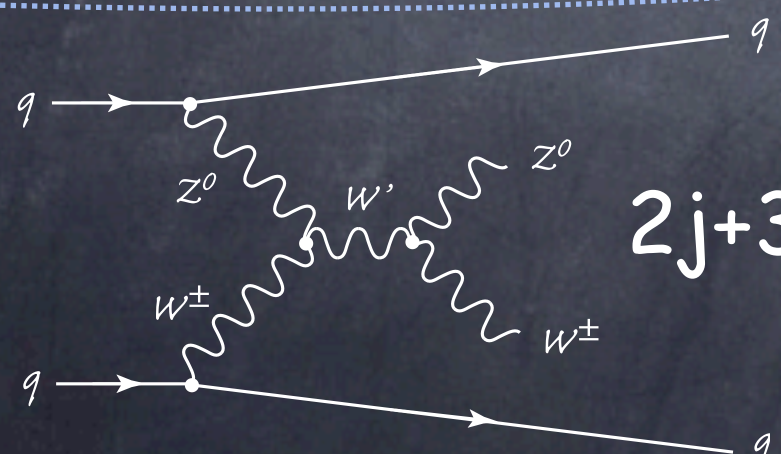


$$g_{WW'Z} \leq \frac{g_{WWZ} M_Z^2}{\sqrt{3} M_{W'} M_W} \qquad \Gamma(W' \to WZ) \sim \frac{\alpha M_{W'}^3}{144 s_w^2 M_W^2}$$

a narrow and light resonance

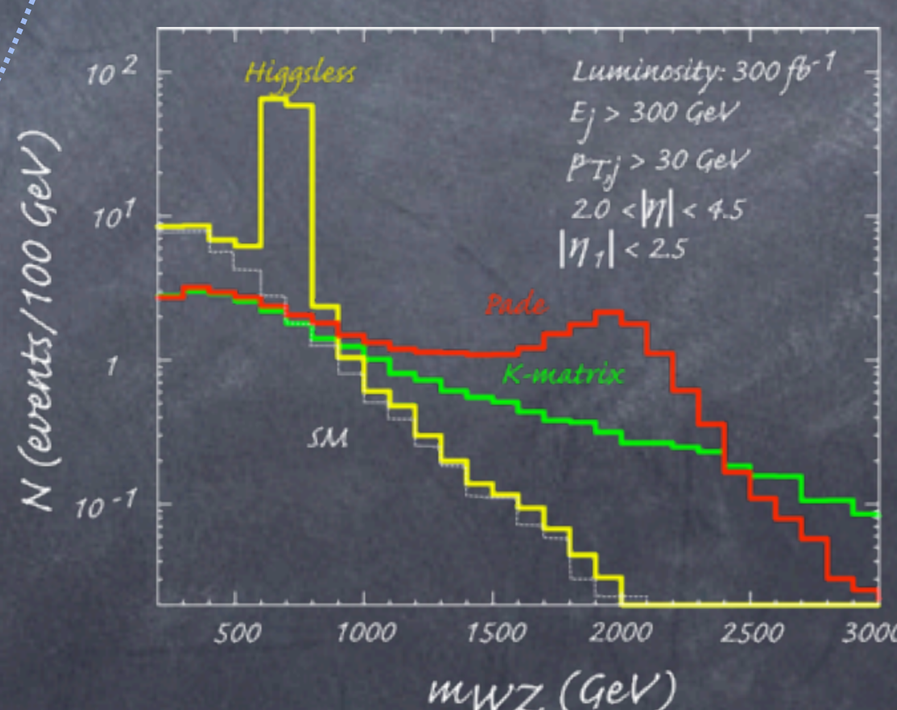
no resonance in WZ for SM/MSSM

W' production



2j+3l+$E\!\!\!/_T$

VBF (LO) dominates over DY since couplings of q to W' are reduced



Number of events at the LHC, 300 fb^{-1}

discovery reach @ LHC (10 events)

$550\ \text{GeV} \to 10\ \text{fb}^{-1}$

$1\ \text{TeV} \to 60\ \text{fb}^{-1}$

should be seen within one/two year

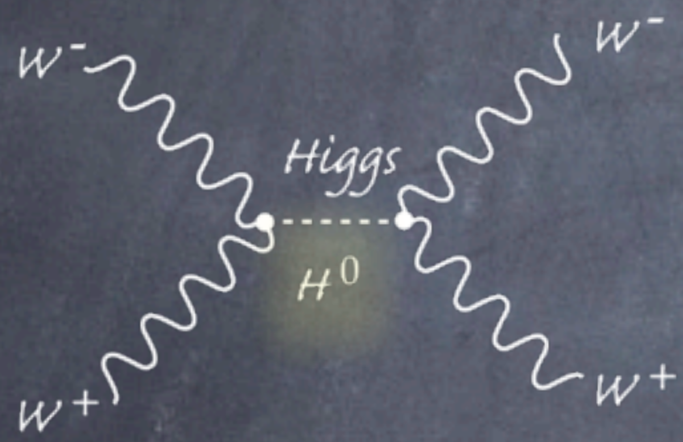
Composite Higgs Models

SM Higgs as a peculiar scalar resonance

A single scalar degree of freedom with no charge under $SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{\rm EWSB} = a\,\frac{v}{2}\,h\,{\rm Tr}\left(D_\mu\Sigma^\dagger D_\mu\Sigma\right) + b\,\frac{1}{4}\,h^2\,{\rm Tr}\left(D_\mu\Sigma^\dagger D_\mu\Sigma\right)$$

'a' and 'b' are arbitrary free couplings



$$\mathcal{A} = \frac{1}{v^2}\left(s - \frac{a^2\,s^2}{s-m_h^2}\right)$$

growth cancelled for

a = 1

restoration of

perturbative unitarity

For b = a^2: perturbative unitarity also maintained in inelastic channels

'a=1' & 'b=1' define the SM Higgs

$\mathcal{L}_{\rm mass} + \mathcal{L}_{\rm EWSB}$ can be rewritten as $D_\mu H^\dagger D_\mu H$

$$H = \frac{1}{\sqrt{2}}\,e^{i\sigma^a\pi^a/v}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

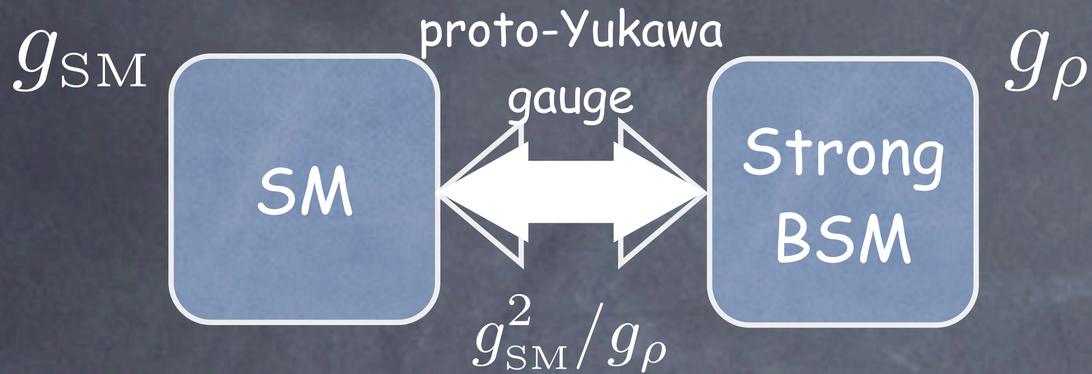
h and π^a (ie W_L and Z_L) combine to form a linear representation of $SU(2)_L \times U(1)_Y$

Deformations of the SM

- Why a single Higgs?
  - why not? Simplicity argument.
  - more Higgs doublets could be dangerous:
    - more complicated vacuum structure
    - possible Higgs-mediated FCNCs
  - triplet Higgs etc: custodial breaking ⇨ small vevs only
- A composite Higgs seems a “soft” deformation of the SM

How to obtain a light composite Higgs?

Higgs=Pseudo-Goldstone boson of the strong sector



$m_{\rm Higgs}=0$ when $g_{\rm SM}=0$

global symmetry → G/H ← residual global symmetry

3 scales:

} not directly accessible to LC

indirect probes

strong sector broadly characterized by 2 parameters

m_ρ = mass of the resonances

g_ρ = coupling of the strong sector or decay cst of strong sector $f = \frac{m_\rho}{g_\rho}$

Continuous interpolation between SM and TC

$$\xi = \frac{v^2}{f^2} = \frac{(\text{weak scale})^2}{(\text{strong coupling scale})^2}$$

$\xi = 0$

SM limit

all resonances of strong sector, except the Higgs, decouple

$\xi = 1$

Technicolor limit

Higgs decouple from SM; vector resonances like in TC

Composite Higgs

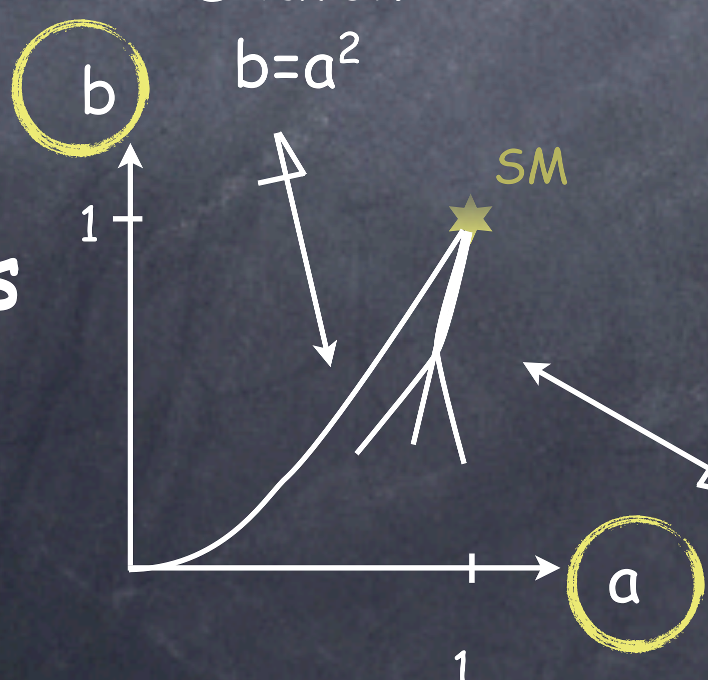
vs.

SM Higgs

Dilaton

Dilaton

$b=a^2$



$$\mathcal{L}_{\rm EWSB} = \left(a\,\frac{v}{2}\,h \; + b\,\frac{1}{4}\,h^2 \right) \mathrm{Tr}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right)$$

Composite Higgs

universal behavior for large f

a=1-v/2f b=1-2v/f

Testing the composite nature of the Higgs?

if LHC sees a Higgs and nothing else*:
is it elementary or composite?

¿¿¿ evidence for fine-tuning & string landscape ???

¿¿¿ Higgs forces have a secret hidden gauge origin ???

- Model-dependent: production of resonances at m_ρ
- Model-independent: study of Higgs properties & W scattering
  - strong WW scattering
  - strong HH production
  - Higgs anomalous coupling
  - anomalous gauge bosons self-couplings

* a likely possibility that precision data seems to point to,
at least in strongly coupled models

What distinguishes a composite Higgs?

Giudice, Grojean, Pomarol, Rattazzi '07

$$\mathcal{L} \supset \frac{c_H}{2f^2} \partial^\mu \left(|H|^2\right) \partial_\mu \left(|H|^2\right) \qquad c_H \sim \mathcal{O}(1)$$

$$U = e^{i\begin{pmatrix} & H/f \\ H^\dagger/f & \end{pmatrix}} U_0$$

$$f^2 \,\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) = |\partial_\mu H|^2 + \frac{\sharp}{f^2}\left(\partial |H|^2\right)^2 + \frac{\sharp}{f^2}|H|^2 \,|\partial H|^2 + \frac{\sharp}{f^2}\left|H^\dagger \partial H\right|^2$$

Anomalous Higgs Couplings

Giudice, Grojean, Pomarol, Rattazzi '07

$$\mathcal{L} \supset \frac{c_H}{2f^2} \partial^\mu \left(|H|^2\right) \partial_\mu \left(|H|^2\right) \qquad c_H \sim \mathcal{O}(1)$$

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \Longrightarrow \mathcal{L} = \frac{1}{2}\left(1 + c_H \frac{v^2}{f^2}\right) (\partial^\mu h)^2 + \ldots$$

Modified Higgs propagator ~ Higgs couplings rescaled by $\frac{1}{\sqrt{1+c_H \frac{v^2}{f^2}}} \sim 1 - c_H \frac{v^2}{2f^2} \equiv 1 - \xi/2$

SILH Effective Lagrangian

(strongly-interacting light Higgs)

Giudice, Grojean, Pomarol, Rattazzi '07

- extra Higgs leg: H/f
- extra derivative: ∂/m_ρ

Genuine strong operators (sensitive to the scale f)

$$\frac{c_H}{2f^2}\left(\partial_\mu\left(|H|^2\right)\right)^2$$

$$\frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)^2$$

custodial breaking

$$\frac{c_y y_f}{f^2}|H|^2 \bar{f}_L H f_R + \text{h.c.}$$

$$\frac{c_6\lambda}{f^2}|H|^6$$

Form factor operators (sensitive to the scale m_ρ)

$$\frac{ic_W}{2m_\rho^2}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)(D^\nu W_{\mu\nu})^i$$

$$\frac{ic_B}{2m_\rho^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)(\partial^\nu B_{\mu\nu})$$

$$\frac{ic_{HW}}{m_\rho^2}\frac{g_\rho^2}{16\pi^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu}$$

$$\frac{ic_{HB}}{m_\rho^2}\frac{g_\rho^2}{16\pi^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$$

minimal coupling: $h \to \gamma Z$

loop-suppressed strong dynamics

$$\frac{c_\gamma}{m_\rho^2}\frac{g_\rho^2}{16\pi^2}\frac{g^2}{g_\rho^2}H^\dagger H B_{\mu\nu}B^{\mu\nu}$$

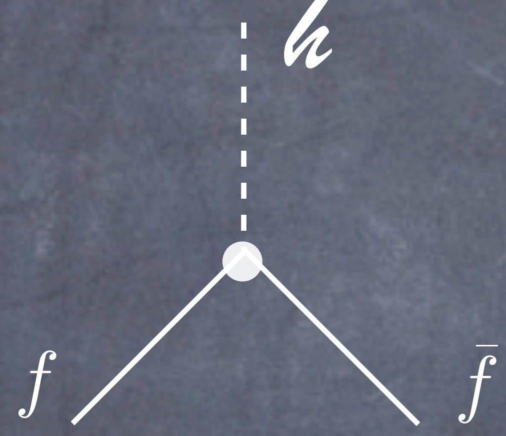
$$\frac{c_g}{m_\rho^2}\frac{g_\rho^2}{16\pi^2}\frac{y_t^2}{g_\rho^2}H^\dagger H G^a_{\mu\nu}G^{a\mu\nu}$$

Goldstone sym.

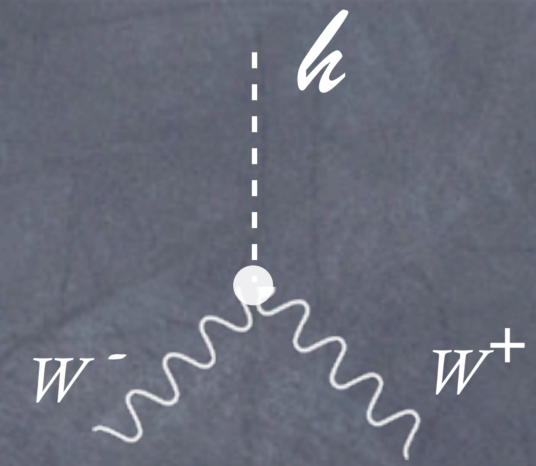
Higgs anomalous couplings

Lagrangian in unitary gauge

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \left(-\frac{m_H^2}{2v}(c_6 - 3c_H/2)h^3 + \frac{m_f}{v}\bar{f}f(c_y + c_H/2)h - c_H\frac{m_W^2}{v}hW_\mu^+W^{-\mu} - c_H\frac{m_Z^2}{v}hZ_\mu Z^\mu \right)\frac{v^2}{f^2} + \ldots$$



$$g_{SM}\left(1-(c_y+c_H/2)v^2/f^2\right)$$



$$g_{SM}\left(1-c_H v^2/f^2\right)$$

⇘ ⇙

$$\Gamma\left(h\to f\bar{f}\right)_{\rm SILH} = \Gamma\left(h\to f\bar{f}\right)_{\rm SM}\left[1-\left(2c_y+c_H\right)v^2/f^2\right]$$

$$\Gamma\left(h\to gg\right)_{\rm SILH} = \Gamma\left(h\to gg\right)_{\rm SM}\left[1-\left(2c_y+c_H\right)v^2/f^2\right]$$

Note: same Lorentz structure as in SM. Not true anymore if form factor ops. are included

Higgs anomalous couplings for large v/f

The SILH Lagrangian is an expansion for small v/f
The 5D MCHM gives a completion for large v/f

$$m_W^2 = \frac{1}{4} g^2 f^2 \sin^2 v/f \quad \Rrightarrow \quad g_{hWW} = \sqrt{1-\xi}\, g_{hWW}^{\mathrm{SM}}$$

Fermions embedded in spinorial of SO(5)

$$m_f = M \sin v/f$$

$$\Downarrow$$

$$g_{hff} = \sqrt{1-\xi}\, g_{hff}^{\mathrm{SM}}$$

universal shift of the couplings
no modifications of BRs

Fermions embedded in 5+10 of SO(5)

$$m_f = M \sin 2v/f$$

$$\Downarrow$$

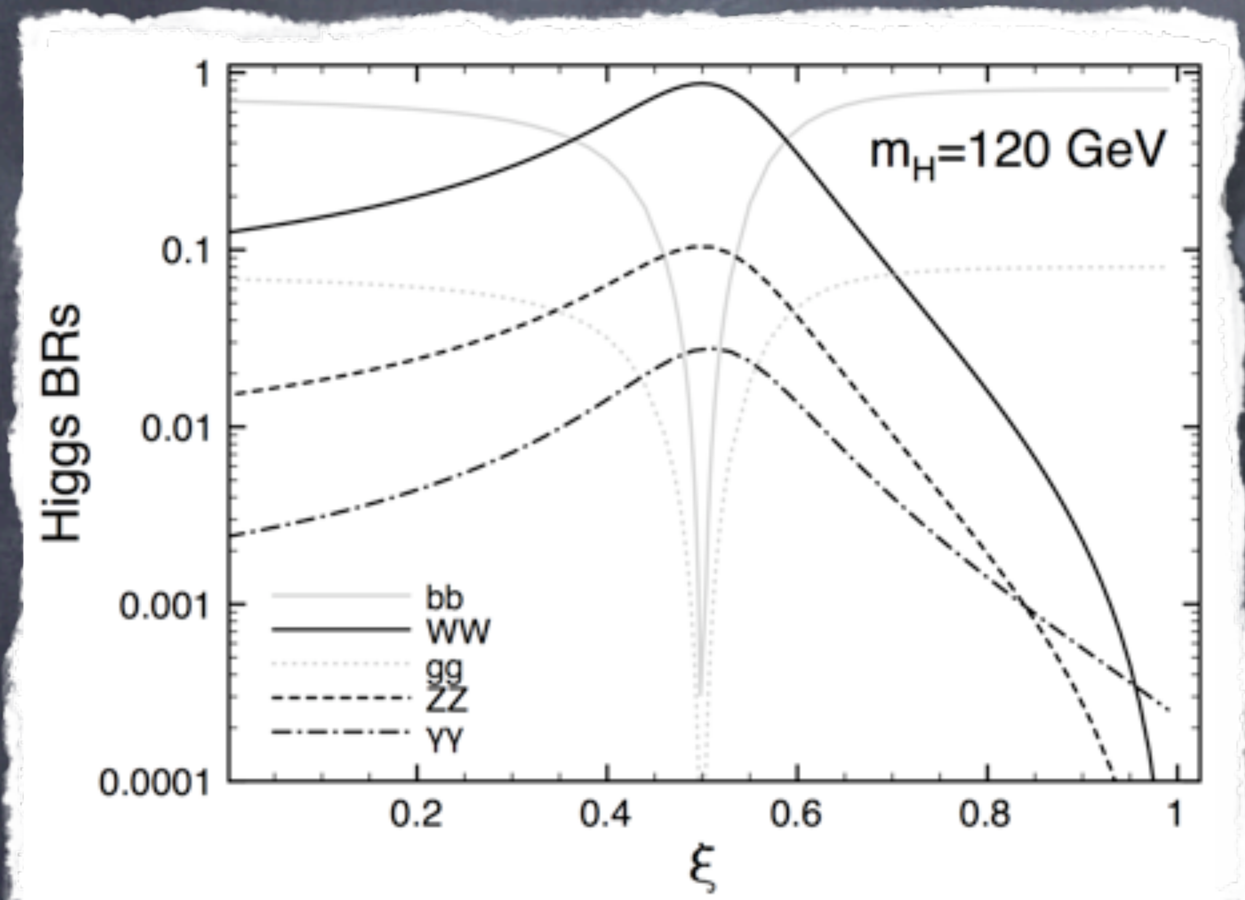
$$g_{hff} = \frac{1-2\xi}{\sqrt{1-\xi}}\, g_{hff}^{\mathrm{SM}}$$

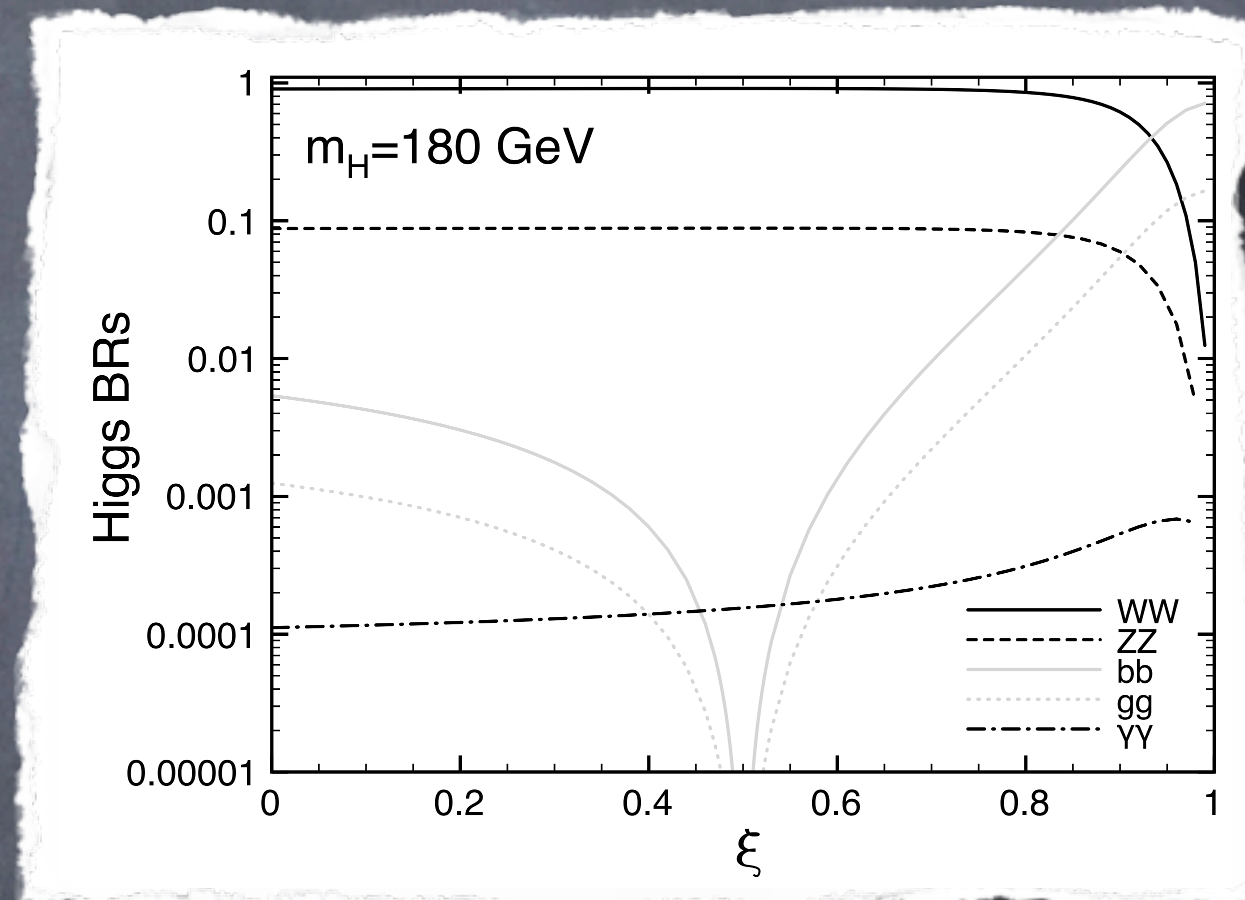
BRs now depends on v/f

$$(\ \xi = v^2/f^2\ )$$

Higgs BRs

Fermions embedded in 5+10 of SO(5)





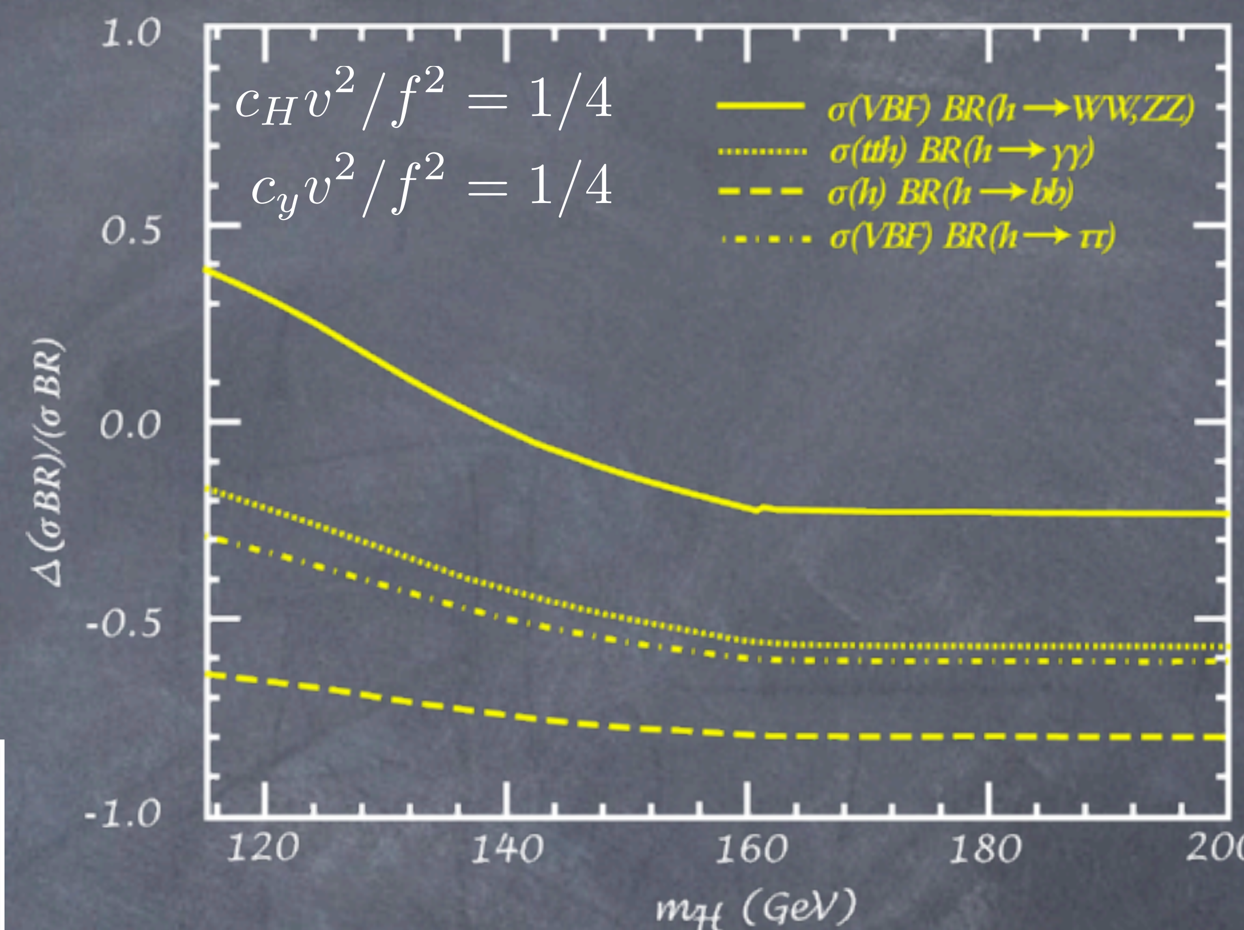
h→WW can dominate even for low Higgs mass

BRs remain SM like except for very large values of v/f

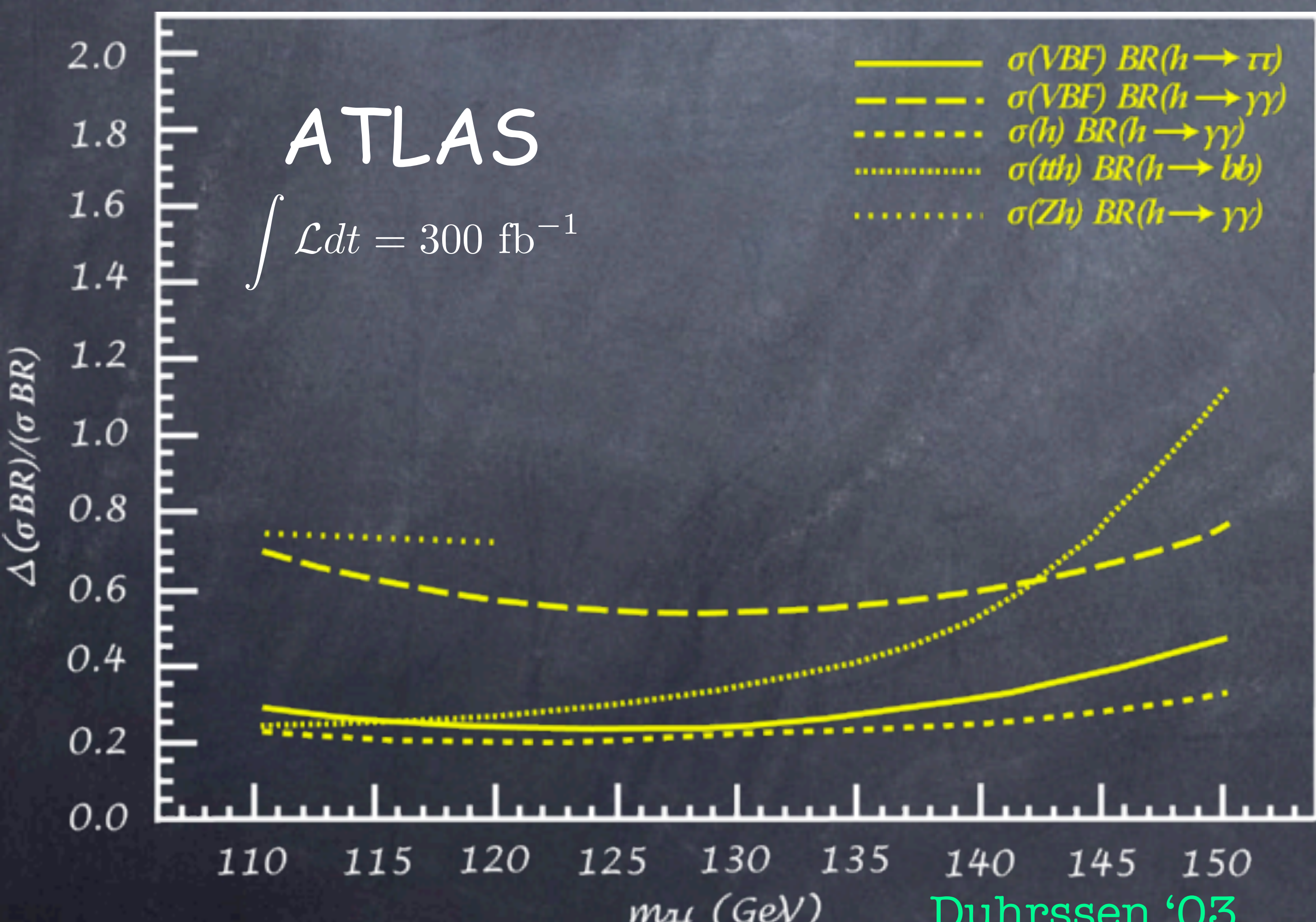
Higgs anomalous couplings @ LHC

$$\Gamma\left(h \to f\bar{f}\right)_{\rm SILH} = \Gamma\left(h \to f\bar{f}\right)_{\rm SM}\left[1-\left(2c_y+c_H\right)v^2/f^2\right]$$

$$\Gamma\left(h \to gg\right)_{\rm SILH} = \Gamma\left(h \to gg\right)_{\rm SM}\left[1-\left(2c_y+c_H\right)v^2/f^2\right]$$



observable @ LHC?



Duhrssen '03

LHC can measure

$$c_H\frac{v^2}{f^2},\quad c_y\frac{v^2}{f^2}$$

up to 0.2-0.4

i.e. $4\pi f \sim 5-7\ \text{TeV}$

(ILC could go to few % ie test composite Higgs up to $4\pi f \sim 30\ \text{TeV}$)

Triple gauge boson couplings (TGC) @ LC

$$\mathcal{L}_V = -ig\cos\theta_W g_1^Z Z^\mu \left(W^{+\nu}W^-_{\mu\nu} - W^{-\nu}W^+_{\mu\nu}\right) - ig\left(\cos\theta_W \kappa_Z Z^{\mu\nu} + \sin\theta_W \kappa_\gamma A^{\mu\nu}\right) W^+_\mu W^-_\nu$$

TGC are generated by heavy resonances

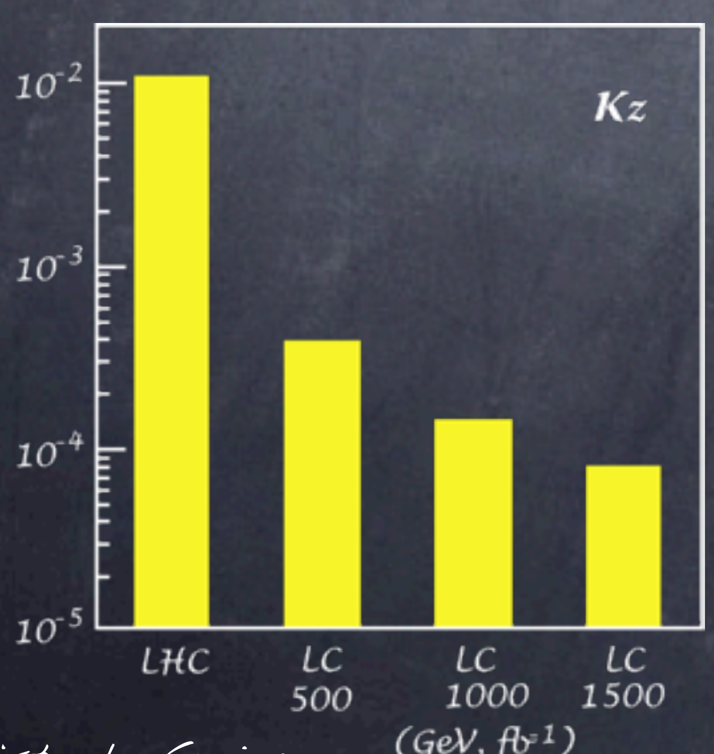
$$g_1^Z = \frac{m_Z^2}{m_\rho^2} c_W \qquad \kappa_\gamma = \frac{m_W^2}{m_\rho^2}\left(\frac{g_\rho}{4\pi}\right)^2 (c_{HW} + c_{HB}) \qquad \kappa_Z = g_1^Z - \tan^2\theta_W \kappa_\gamma$$

@ LHC 100fb^{-1} $\quad g_1^Z \sim 1\% \qquad \kappa_\gamma \sim \kappa_Z \sim 5\%$

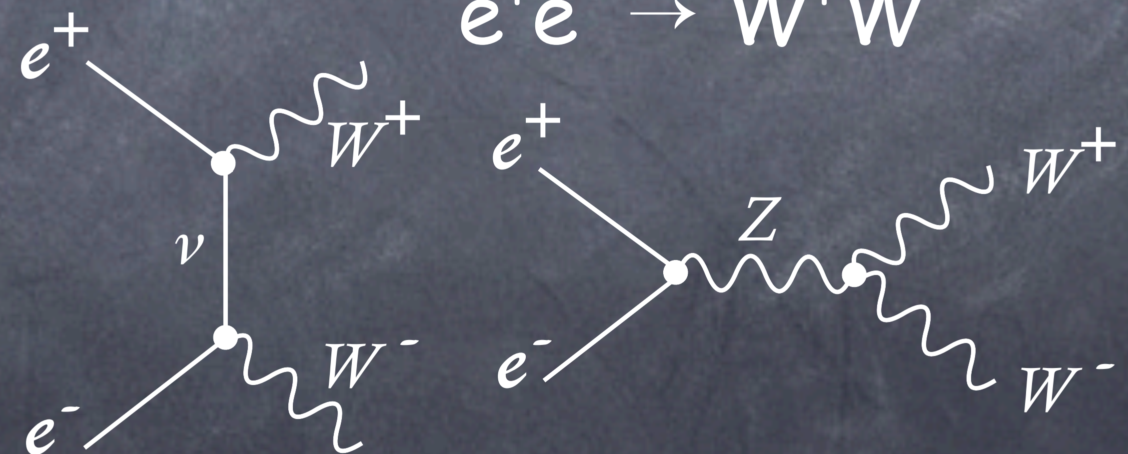
sensitive to resonance up to m_ρ~800 GeV

not competitive with the measure of S at LEPII

@ ILC



$e^+e^- \to W^+W^-$



0.1% accuracy $\Longrightarrow$ sensitive to resonance up to m_ρ~8TeV

T. Abe et al, Snowmass '01

Strong WW scattering

Giudice, Grojean, Pomarol, Rattazzi '07

$$\mathcal{L} \supset \frac{c_H}{2f^2} \partial^\mu \left(|H|^2\right) \partial_\mu \left(|H|^2\right) \qquad c_H \sim \mathcal{O}(1)$$

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \Longrightarrow \mathcal{L} = \frac{1}{2}\left(1 + c_H \frac{v^2}{f^2}\right) (\partial^\mu h)^2 + \dots$$

Modified Higgs propagator ~ Higgs couplings rescaled by $\frac{1}{\sqrt{1 + c_H \frac{v^2}{f^2}}} \sim 1 - c_H \frac{v^2}{2f^2} \equiv 1 - \xi/2$

$$= -(1-\xi) g^2 \frac{E^2}{M_W^2}$$

no exact cancellation of the growing amplitudes

Even with a light Higgs, growing amplitudes (at least up to m_ρ)

$$\mathcal{A}\left(W_L^a W_L^b \to W_L^c W_L^d\right) = \mathcal{A}(s,t,u)\delta^{ab}\delta^{cd} + \mathcal{A}(t,s,u)\delta^{ac}\delta^{bd} + \mathcal{A}(u,t,s)\delta^{ad}\delta^{bc}$$

$$\mathcal{A}_{\text{LET}}(s,t,u) = \frac{s}{v^2} \Longrightarrow \mathcal{A}_\xi = \xi\, \mathcal{A}_{\text{LET}}$$

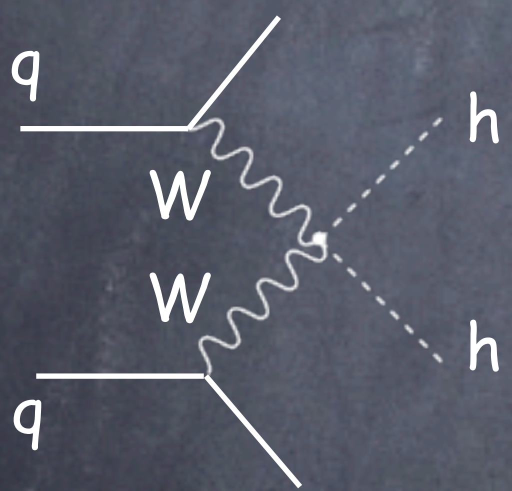
LET=SM-Higgs

Strong Higgs production

O(4) symmetry between W_L, Z_L and the physical Higgs

strong boson scattering ⇔ strong Higgs production

$$\mathcal{A}\left(Z_L^0 Z_L^0 \to hh\right) = \mathcal{A}\left(W_L^+ W_L^- \to hh\right) = \frac{c_H s}{f^2}$$



signal:

- $hh \to bbbb$
- $hh \to 4W \to 3l^{\pm}3\nu$ + jets

More complicated final states than for $WW \to WW$,
smaller BRs,
but no T polarization pollution

Strong Higgs production: (3L+jets) analysis

Contino, Grojean, Moretti, Piccinini, Rattazzi 'in progress

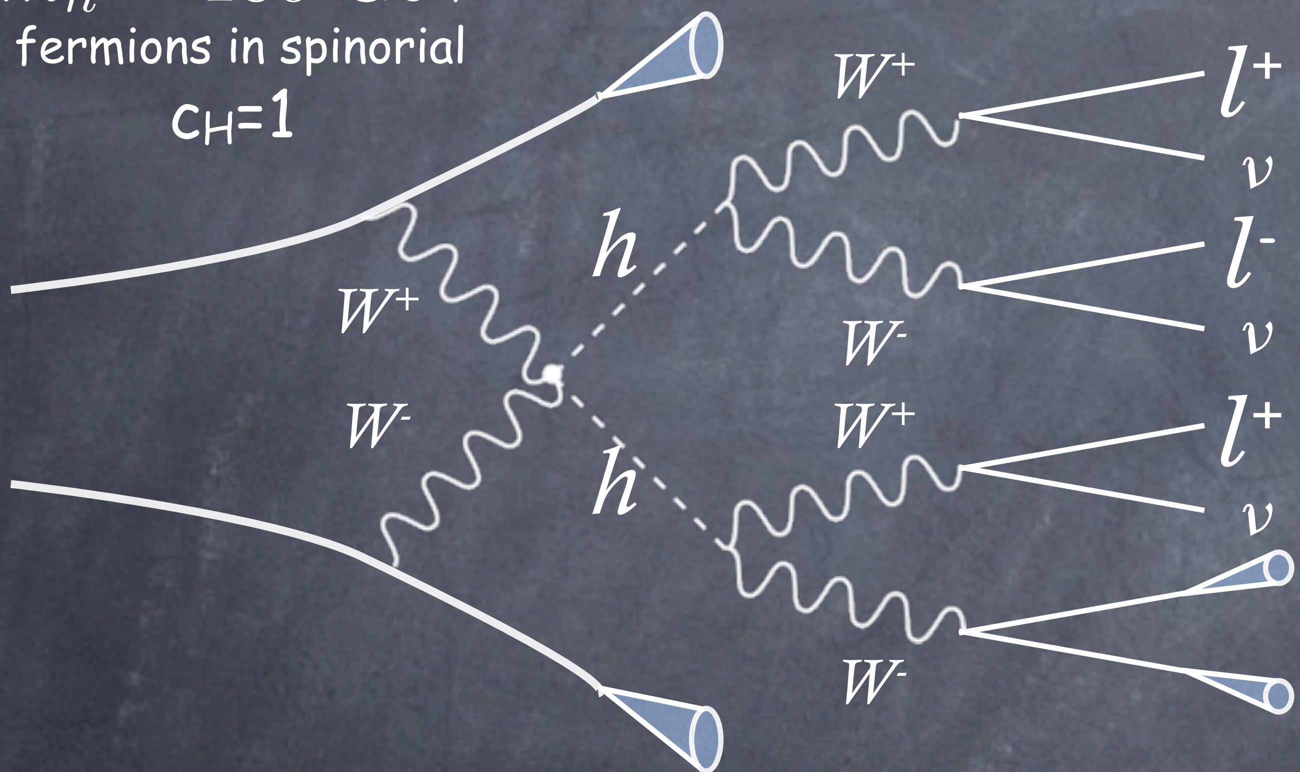
strong boson scattering ⇔ strong Higgs production

$$\mathcal{A}\left(Z_L^0 Z_L^0 \to hh\right) = \mathcal{A}\left(W_L^+ W_L^- \to hh\right) = \frac{c_H s}{f^2}$$

$m_h = 180\ \mathrm{GeV}$

fermions in spinorial

c_H=1



acceptance cuts

jets	leptons
$p_T \geq 30\ \mathrm{GeV}$	$p_T \geq 20\ \mathrm{GeV}$
$\delta R_{jj} > 0.7$	$\delta R_{lj(ll)} > 0.4(0.2)$
$\lvert\eta_j\rvert \leq 5$	$\lvert\eta_j\rvert \leq 2.4$

Dominant backgrounds: Wll4j, $t\bar{t}$W2j, $t\bar{t}$2W, 3W4j...

forward jet-tag, back-to-back lepton, central jet-veto

v/f	1	$\sqrt{.8}$	$\sqrt{.5}$
significance (300 fb^{-1})	4.0	2.9	1.3
luminosity for 5σ	450	850	3500

⇚ good motivation to SLHC

Conclusions

"theorists are getting cold feet" J. Ellis

"they have done their best to predict the possible and impossible"
G. Giudice

I guess, during these lectures, I gave you a flavour of what the impossible could be!

LHC is prepared to discover the "Higgs"

collaboration EXP-TH is important to make sure
e.g. that no unexpected physics is missed (triggers, cuts...)
and in this regards, approaches like "unparticle" or "hidden valleys" might be useful.

Thank you for your attention

and good luck for your PhD.